

COMPOSED BY SADIA ALI SADI

	Discrete Mathematics MTH 202 Semester Fall 2004	Total Marks: 40 Time: 60 min
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Instructions

Please read the following instructions carefully before attempting any questions:

1. The duration of this examination is 60 minutes.
2. This examination is closed book, closed notes, closed neighbors
3. Answer all questions.
4. Do not ask any questions about the contents of this examination from anyone and if you feel that there is something wrong with any question then make the best assumptions which you think and then answer that question
5. Calculator is allowed

Question No: 1

Marks : 2

If $R = \{(a, a), (b, b), (c, c)\}$ is a relation on the set $A = \{a, b, c\}$ Then R is

- a) Symmetric only.
- b) Symmetric and reflexive only.
- c) Reflexive only.
- d) **Equivalence**

Question No: 2

Marks : 2

The negation of the implication "If P is a square then P is a rectangle" is

1. If P is not a square then P is not a rectangle
2. P is not a square and P is a rectangle
3. **P is a square and P is not a rectangle.**
4. None of the above

Question No: 3**Marks : 2**

Identify the false statement

1. $\emptyset \subseteq \emptyset$
2. $\{ \emptyset \} \subseteq \{ \emptyset \}$
3. If A and B are two sets $A \subseteq B$ and $B \subseteq A$ then $A = B$.
4. Two sets are disjoint if their intersection is empty set.
5. $A \cap A^c = U$

Question No: 4**Marks : 2**

Let A be a set containing 3 elements then the total number of relations from A to A

1. 2^{*9}
2. 2^9
3. n^n
4. 2^{n^2}

Question No: 5**Marks : 2**Let $A = \{1,2,3\}$ and $B = \{2,3,4,5\}$ then

1. $A = B$.
2. A is a subset of B.
3. A is improper subset of B.
4. **Both 2 and 3.**

Question No: 6**Marks : 5**

Construct truth table for the following compound

$$(p \rightarrow q) \leftrightarrow (p \wedge q)$$

solution:

p	q	$p \rightarrow q$	$p \wedge q$	$(p \rightarrow q) \leftrightarrow (p \wedge q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F

Marks distribution

- 1 marks for column 1
- 1 marks for column 2
- 1 marks for column 3
- 1 marks for column 4
- 1 marks for column 5

Question No: 7**Marks : 5**

What are the contra positive , the inverse , and the converse of the

"If you have flu then you will miss the final examination"

Solution:

- Let p = you have flu
 Q = you will miss the final examination

Contra positive

$$\sim q \rightarrow \sim p$$

If you will not miss the final examination then you have no flu.

Inverse

$$\sim p \rightarrow \sim q$$

If you have no flu then you will not miss the final examination

Converse

$$q \rightarrow p$$

If you will miss the final examination then you have flu.

Marks distribution

2 marks for contra positive

2 marks for the inverse

1 marks for the converse

Question No: 8**Marks : 5**

Determine whether the R on the set of integers is transitive, where $x, y \in \mathbb{Z}$ if and only if $2|(x - y)$

Solution:

To show that R is reflexive, it is necessary to show that

For all $x, y, z \in \mathbb{Z}$ if $x R y, y R z$ then $x R z$

By definition of R this means that

For all $x, y, z \in \mathbb{Z}$, if $2|(x - y), 2|(y - z)$ then $2|(x - z)$

Now by definition of "divides"

$$2|(x - y)$$

$$x - y = 2k \quad \text{for some integer } k$$

Now by definition of "divides"

$$2|(y - z)$$

$$y - z = 2m \quad \text{for some integer } m$$

$$(x - y) + (y - z) = 2k + 2m$$

$$x - z = 2(k + m)$$

Since k and m are integers then $k + m$ is also an integer

Let $k + m = s$ for some integer s

$$x - z = 2s$$

This implies that $2|(x - z)$

It follows that R is Transitive

Marks distribution

2 marks for the definition

2 marks for the calculations

1 marks for the required result

Let A, B and C be sets. Show that

$$(A \cup (B \cap C))^c = (C^c \cup B^c) \cap A^c$$

By using set identities.

Solution:

$$\begin{aligned} & (A \cup (B \cap C))^c \\ &= A^c \cap (B \cap C)^c \text{ By De Morgan's law} \\ &= A^c \cap (B^c \cup C^c) \text{ By De Morgan's law} \\ &= (B^c \cup C^c) \cap A^c \text{ By commutative law for intersection} \\ &= (C^c \cup B^c) \cap A^c \text{ By commutative law for union} \end{aligned}$$

Marks distribution

2 marks for applying the De Morgan's law

1 marks for applying commutative law

2 marks for applying commutative law of union

If the 3rd element of an arithmetic series is -16 and the 20th term is -46. Then find the 10th term?

Solution:

$$a_3 = -16$$

$$a_{20} = -46$$

$$a_{10} = ?$$

Using the formula

$$a_n = a_1 + (n-1)d$$

Putting the values in the above formula we get

$$-16 = a_1 + (3-1)d$$

$$-46 = a_1 + (20-1)d$$

Solving the above two equations we have

$$d = \frac{-30}{17}$$

$$a_1 = \frac{-212}{17}$$

$$a_{10} = a_1 + (10-1)d$$

$$a_{10} = \frac{-212}{17} + 9\left(\frac{-30}{17}\right)$$

$$a_{10} = \frac{-212 - 270}{17}$$

$$a_{10} = \frac{-482}{17}$$

Marks distribution

- 1 marks for using formula
- 1 marks for substituting values in the formula
- 1 marks for solving the equation
- 1 marks for finding the correct values of d and a1
- 1 marks for the answer

Question No: 11**Marks : 5**

Suppose that the relation R on a set is represented by the

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Is R reflexive, symmetric, and/or anti symmetric?

Solution:

Since all the diagonal elements of this matrix are equal to 1, R is reflexive. Moreover, since M_R is symmetric because $M_R = M_R^t$, it follows that R is symmetric. But R is not anti symmetric.

Marks distribution

- 2 marks for reflexive
- 2 marks for symmetric
- 1 marks for anti symmetric