Laws of Logic.

1. ***Cumulative Law***

$$p \wedge q = q \wedge p$$
$$p \vee q = q \vee p$$

2. ***Associative Law***

$$(p \wedge q) \wedge r = p \wedge (q \wedge r)$$
$$(p \vee q) \vee r = p \vee (q \vee r)$$

3. ***Distributive Law***

$$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$
$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$$

4. ***Idempotent Law***

$$p \wedge p = p$$
$$p \vee p = p$$

5. ***De-Morgans Law***

$$\sim (p \wedge q) = \sim p \vee \sim p$$
$$\sim (p \vee q) = \sim p \wedge \sim p$$

6. ***Absorption Law***

$$p \vee (p \wedge q) = p$$
$$p \wedge (p \vee q) = p$$

7. ***Identity Law***

$$p \wedge t = p$$
$$p \vee c = p$$

8. ***Universal Bond Law***

$$p \vee t = t$$
$$p \wedge c = c$$

9. ***Negation Law***

$p \vee \sim p = t$ *(Tautology)*
$p \wedge \sim p = c$ *(Contradiction)*

10. ***Negation of t and c***

$$\sim t = c$$
$$\sim c = t$$

11. ***Double Negation Law***

$$\sim (\sim p) = p$$

Set Identities.

Let A,B, and C be subsets of a Universal set U. then

1. Idempotent Law:

a. $A \cup A = A$, b. $A \cap A = A$

2. Commutative Law:

a. $A \cup B = B \cup A$, b. $A \cap B = B \cap A$

3. Associative Law:

a. $A \cup (B \cup C) = (A \cup B) \cup C$,

b. $A \cap (B \cap C) = (A \cap B) \cap C$

4. Distributive Law:

a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$,

b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. Identity Laws:

a. $A \cup \varnothing = A$, b. $A \cap \varnothing = \varnothing$

c. $A \cup U = U$, d. $A \cap U = A$

6. Complement Laws:

a. $A \cup A^C = U$, b. $A \cap A^C = \varnothing$

c. $U^C = \varnothing$, d. $\varnothing^C = U$

7. Double Complement Law:

a. $(A^C)^C = A$,

8. De-Morgan's Law:

a. $(A \cup B)^C = A^C \cap B^C$, b. $(A \cap B)^C = A^C \cup B^C$

9. Alternate Representation for Set Difference:

$A - B = A \cap B^C$

10. Subset Laws:

a. $A \cup B \subseteq C$ iff $A \subseteq C$ and $B \subseteq C$

c. $C \subseteq A \cap B$ iff $C \subseteq A$ and $C \subseteq B$

11. Absorption Laws:

a. $A \cup (A \cap B) = A$, b. $A \cap (A \cup B) = A$

Implications(Conditional)& Bi-Conditional

Implication (Conditional) $\rightarrow$

1- a. $p \rightarrow q \equiv \sim q \rightarrow \sim p$, **b.** $p \rightarrow q \equiv \sim p \vee q$

2- if $p \rightarrow q$, then its ***inverse*** is $\sim p \rightarrow \sim q$

3- if $p \rightarrow q$, then its ***converse*** is $q \rightarrow p$

$\rightarrow$ operator is not a commutative coz $p \rightarrow q \neq q \rightarrow p$

4- if $p \rightarrow q$, then its ***contra positive*** is $\sim q \rightarrow \sim p$

so $p \rightarrow q \equiv \sim q \rightarrow \sim p$, so implication is equivalent to its contra positive.

Bi-Conditional $\leftrightarrow$

1- $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

2- $\sim p \leftrightarrow q \equiv p \leftrightarrow \sim q$

Laws of Logic.

1- Commutative Law : $p \leftrightarrow q \equiv q \leftrightarrow p$

2- Implication Law: $p \rightarrow q \equiv \sim p \vee q$

$\equiv \sim (p \wedge \sim q)$

3- Exportation Law: $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$

4- Equivalence: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

5- Reductio ad absurdum: $p \rightarrow q \equiv (p \wedge \sim q) \rightarrow c$

1.

Solution:

f: R $\rightarrow$R

g: R $\rightarrow$R

f(x) = x^2 g(x) = 3x + 1

(fog)(x) = f [g(x)]

= f [3x + 1]

= $(3x + 1)^2$

= $9x^2 + 6x + 1$

(gof)(x) = g [f(x)]

= g [x^2]

= $3(x^2) + 1$

= $3x^2 + 1$

We observe that fog $\neq$gof, that is the commutative law does not hold for the composition of functions.

2.

Solution:

f: R $\rightarrow$R

f(x) = $x^2 - 3x + 2$

(fof)(x) = f [f(x)]

= f $(x^2 - 3x + 2)$

= $(x^2 - 3x + 2)^2 - 3(x^2 - 3x +2) + 2$

= $x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2$

$- 3x^2 + 9x - 6 + 2$

= $x^4 - 6x^3 + 10x^2 - 3x$.

3.

f: R → R defined by $f(x) = x^2 + 3x + 1$

g: R → R defined by $g(x) = 2x - 3$

i. $fog(x) = f[g(x)]$

$= f[2x - 3]$

$= (2x - 3)^2 + 3(2x - 3) + 1$

$= 4x^2 - 12x + 9 + 6x - 9 + 1$

$= 4x^2 - 6x + 1$

ii. $gof(x) = g[f(x)]$

$= g[x^2 + 3x + 1]$

$= 2(x^2 + 3x + 1) - 3$

$= 2x^2 + 6x + 2 - 3$

$= 2x^2 + 6x - 1$

iii. $fof(x) = f[f(x)]$

$= f[x^2 + 3x + 1]$

$= (x^2 + 3x + 1)^2 + 3(x^2 + 3x + 1) + 1$

$= x^4 + 11x^2 + 1 + 6x^3 + 6x + 3x^2 + 9x + 3 + 1$

$= x^4 + 6x^3 + 14x^2 + 15x + 5$

iv. $gog(x) = g[g(x)]$

$= g[2x - 3]$

$= 2(2x - 3) - 3$

$= 4x - 6 - 3$

$= 4x - 9$

4.

f: R → R defined by $f(x) = x + 1$

g: R → R defined by g(x) = x - 1

fog(x) = f [g(x)]

= f (x - 1)

= (x - 1) + 1

= x

gof (x) = g [f(x)]

= g [x + 1]

= (x + 1) -1

= x

∴ fog = gof = I_R

I_R is the identity function.

∴ $I_R(x)$ = x.

5.

f: N → Z_0 defined by f(x) = 2x

g: Z_0 → Q defined by g(x) = 1/x

h: Q → R defined by h(x) = 5x + 2

To verify associativity we have to prove that ho(gof) = (hog)of.

Consider

[ho(gof)](x) = h [(gof) (x)]

= h [g (f (x))]

= h [g (2x)]

= h [1/2x]

= 5 * 1/2x + 2

= 5/2x + 2

$[(hog)of](x) = (hog)[f(x)]$

$= (hog)(2x)$

$= h[g(2x)]$

$= h(1/2x)$

$= 5 * 1/2x + 2$

$= 5/2x + 2$

6.

f: $R \rightarrow R$ is the identity function

$\Rightarrow f(x) = x$

$fof(x) = f[f(x)]$

$= f(x)$

$= x$

$(ff)(x) = f(x) * f(x)$

$= x * x$

$= x^2$

What is the smallest integer N such that

a. $\lceil N/7 \rceil = 5$ b. $\lceil N/9 \rceil = 6$

SOLUTION:

a. $N = 7 \cdot (5 - 1) + 1 = 7 \cdot 4 + 1 = 29$

b. $N = 9 \cdot (6 - 1) + 1 = 9 \cdot 5 + 1 = 46$

EXAMPLE:

Use the Euclidean algorithm to find gcd(330, 156)

Solution:

1.Divide 330 by 156:

This gives $330 = 156 \cdot 2 + 18$

2.Divide 156 by 18:

This gives $156 = 18 \cdot 8 + 12$

3.Divide 18 by 12:

This gives $18 = 12 \cdot 1 + 6$

4.Divide 12 by 6:

This gives $12 = 6 \cdot 2 + 0$

Hence gcd(330, 156) = 6.