

**FINAL TERM EXAMINATION**  
**Fall 2008**  
**(Session - 1)**

# Calculus & Analytical Geometry-I Guls

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## UPDATED VERSION

**Exclusive thanks to Mahar Azahar (Lodhran)**

**Question No: 1 ( Marks: 1 ) - Please choose one**

If  $y = f(x)$  then the average rate of change of  $y$  with respect to  $x$  over the interval  $[x_0, x_1]$  is the ..... Joining the points  $(x_0, f(x_0))$  and  $(x_1, f(x_1))$  on the graph of  $f$

- ▶ Slope of the secant line
- ▶ Slope of tangent line
- ▶ Secant line
- ▶ none of these

**Question No: 2 ( Marks: 1 ) - Please choose one**

Natural domain of  $\frac{(x^2 - 4)}{(x - 2)}$  is

- ▶  $(-\infty, 2) \cup (2, +\infty)$
- ▶  $(-\infty, 2)$
- ▶  $(-\infty, 0)$
- ▶ None of these

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**Question No: 3 ( Marks: 1 ) - Please choose one**

The equation  $(x+4)^2 + (y-1)^2 = 6$  represents a circle having center at ..... and radius .....

- $(-4, 1), \sqrt{6}$
- $(-4, 1), 6$
- $(-4, -1), \sqrt{6}$
- None of these

**Question No: 4 ( Marks: 1 ) - Please choose one**

The series  $\sum u_k$  be a series with positive terms and suppose that if  $\rho > 1$  then the series

$$\rho = \lim_{k \rightarrow \infty} \sqrt[k]{u_k} = \lim_{k \rightarrow \infty} (u_k)^{\frac{1}{k}}$$

- Converges
- Diverges
- May converge or diverge
- None of these

**Question No: 5 ( Marks: 1 ) - Please choose one**

The series  $\sum u_k$  and  $\sum v_k$  are convergent series then  $(\sum u_k + \sum v_k)$  and  $(\sum u_k - \sum v_k)$  will be .....and.....

- Convergent, convergent
- Divergent, divergent
- Convergent, divergent
- Divergent, convergent

**Question No: 6 ( Marks: 1 ) - Please choose one**

The notation  $\left\{\frac{1}{2^n}\right\}_{-1}^n$  represents the sequence

$$2, 1, \frac{1}{2}, \frac{1}{4}, \dots$$

▶ 0, 1, 2, 3...

▶  $0, 1, \frac{1}{2}, \frac{1}{4}, \dots$

▶ None of these

**Question No: 7 ( Marks: 1 ) - Please choose one**

\_\_\_\_\_ If  
f is continuous on (a,b] but does not have a limit from the right then the integral

$$\int_a^b f(x) dx = \lim_{l \rightarrow a} \int_l^b f(x) dx$$

defined by

is called ..... Integral

▶ Improper

▶ Proper

▶ None of these

**Question No: 8 ( Marks: 1 ) - Please choose one**

\_\_\_\_\_ An object is displaced 1m by a force of 1N then the work done W is

▶ 2

▶ 0

▶ None of these

▶ **1**

**Question No: 9 ( Marks: 1 ) - Please choose one**

\_\_\_\_\_ If  
f is a smooth function on [a,b] then the arc length L of the curve  $y=f(x)$  from  $x=a$  to  $x=b$  will be

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

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$$L = \int_a^b \sqrt{1 + [f'(x)]} dx$$



$$L = \int_0^a \sqrt{1 + [f'(x)]} dy$$



▶ None of these

**Question No: 10 ( Marks: 1 ) - Please choose one**

\_\_\_\_\_ If  $f$  is a smooth function on  $[0,3]$  then the arc length  $L$  of the curve  $y=f(x)$  from  $x=0$  to  $x=3$  will be

$$L = \int_0^3 \sqrt{1 + [f'(x)]^2} dx$$



$$L = \int_a^b \sqrt{1 + [f'(x)]^2}$$



$$L = \int_0^3 \sqrt{1 + [f'(x)]^2} dy$$



▶ None of these

**Question No: 11 ( Marks: 1 ) - Please choose one**

\_\_\_\_\_ By using cylindrical shell to find volume of the solid when the region  $R$  in the first quadrant enclosed between  $y = 3x$  and  $y = 2x^2$  is revolved about the  $x$ -axis

$$V = \int_0^{\frac{3}{2}} 2\pi x(3x - 2x^2) dx$$



$$V = \int_0^{\frac{3}{2}} x(3x - 2x^2) dx$$



$$V = \int_0^{\frac{3}{2}} 2\pi(3x - 2x^2) dx$$



▶ None of these

**Question No: 12 ( Marks: 1 ) - Please choose one**

By using cylindrical shell to find volume of the solid when the region R in the first quadrant enclosed between  $y = x$  and  $y = x^2$  is revolved about the y-axis is represented by

$$V = \int_0^3 2\pi x(x - x^2) dx$$



$$V = \int_0^1 x(x - x^2) dx$$



$$V = \int_0^1 2\pi(x - x^2) dx$$



▶ None of these

**Question No: 13 ( Marks: 1 ) - Please choose one**

If

$$\int_a^a f(x) dx =$$

a is in the domain of f, then



$$f'(x)$$



$$f(x)$$

▶ 0

▶ None of these

**Question No: 14 ( Marks: 1 ) - Please choose one**

$$\int_0^2 x^2 dx$$

Consider the integral  $\int_0^2 x^2 dx$ , the area on right is bounded by

▶  $y = x^2$ ▶  $x = 2$ ▶  $x = 0$ 

▶ None of these

**Question No: 15 ( Marks: 1 ) - Please choose one**

The series  $1 - 3 + 5 - 7 + 9 - 11$  may written as in sigma notation

$$\sum_{k=0}^{k=5} (-1)^k (2k + 1)$$

▶

$$\sum_{k=1}^{k=5} (-1)^k (2k + 1)$$

▶

$$\sum_{k=1}^{k=5} (2k + 1)$$

▶

▶ None of these

**Question No: 16 ( Marks: 1 ) - Please choose one**

$4^2 + 5^2 + 6^2 + 7^2$  in sigma notation may be represented as

$$\sum_{k=2}^{k=7} k^2$$



$$\sum_{k=2}^{k=7} (k+1)^2$$



$$\sum_{k=4}^{k=7} k^2$$



▶ None of these

**Question No: 17 ( Marks: 1 ) - Please choose one**

\_\_\_\_\_ If  
a function  $f$  is ..... on a closed interval  $[a,b]$ , then  $f$  has both a maximum and minimum value on  $[a,b]$

▶ **Continuous**

▶ Discontinuous

▶ Differentiable

▶ None of these

**Question No: 18 ( Marks: 1 ) - Please choose one**

\_\_\_\_\_ Let  
 $f$  be a function on an interval, and  $x_1$  and  $x_2$  denote the points in that interval, if  
 $f(x_1) < f(x_2)$   
whenever  
 $x_1 < x_2$   
then the we can say that  $f$  is

▶ **Increasing function**

▶ **Decreasing function**

▶ Constant function

▶ None of these

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**Question No: 19 ( Marks: 1 ) - Please choose one**

\_\_\_\_\_ If  
a function satisfies the conditions  
 $f(c)$  is defined

$$\lim_{x \rightarrow c^+} f(x)$$

Exists

$$\lim_{x \rightarrow c^+} f(x) = f(c)$$

Then the function is said to be

► **Continuous at c**

- Continuous from left at c
- Continuous from right at c
- None of these

**Question No: 20 ( Marks: 1 ) - Please choose one**

For a function  $f(x)$  to be continuous on interval  $(a,b)$  the function must be continuous

► **At all point in  $(a,b)$**

- Only at a and b
- At mid point of a and b
- None of these

**Question No: 21 ( Marks: 2 )**

$$a_{n+1} = \frac{1}{3} \left( a_n + \frac{1}{a_n} \right) \text{ for } n \geq 1 \text{ and } a_1 = 2$$

Write down the first two term of the sequence

**Question No: 22 ( Marks: 2 )**

Find the integral of the surface area of the portion of the sphere generated by revolving the

$$y = \sqrt{2-x^2}, 0 \leq x \leq \frac{1}{3}$$

curve

(Note: Just find the integral do not solve the integral)

**Question No: 23 ( Marks: 2 )**



$$\int_2^5 f(x)dx - \int_2^3 f(x)dx = 7, \int_3^4 f(x)dx = 2, \int_5^4 f(x)dx = 5$$

Calculate  $\int_2^5 f(x)dx$  if

$$\int_2^3 f(x)dx = 7, \int_3^4 f(x)dx = 2, \int_5^4 f(x)dx = 5$$

$$\int_2^5 f(x)dx = 7 + 2 - 5 = 4$$

**Question No: 24 ( Marks: 3 )**

the first two Taylor polynomials for  $\ln x$  about  $x=3$

Find

**Question No: 25 ( Marks: 3 )**

the curve  $y = x^{\frac{3}{2}}$ ;  $0 \leq y \leq 2$ , then find the surface area generated by revolving the curve. (But do not evaluate)

Let

**Question No: 26 ( Marks: 3 )**

$$\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{7225}$$

Express the sum

in sigma notation but do not evaluate.

$$\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{7225}$$

$$\sum_{k=1}^{7225} k^3 + 1$$

**Question No: 27 ( Marks: 5 )**

the first four nonzero terms of the Taylor series generated by  $f$  at  $x=a$

Find

$$f(x) = \frac{1}{1-x} \text{ at } x=2$$

**Question No: 28 ( Marks: 5 )**

Evaluate the Definite Integral using the First fundamental Theorem of Calculus

$$\int_0^1 (x^5 - x^3 + 2x) dx$$

Let  $u = (x^5 - x^3 + 2x)$

$$\int_0^1 (u) dx$$

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**Question No: 29 ( Marks: 5 )**

Express the definite integrals as limits (Do not evaluate the integrals)

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \cos x) dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \cos x) dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \lim_{\max \Delta x \rightarrow 0} \sum_{i=1}^n (1 + \cos x) dx$$

pdfelement

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**Question No: 30 ( Marks: 10 )**

Find

the region enclosed by the curves and also find the area

$$y = x^2, y = \sqrt{x}, x = \frac{1}{4}, x = 1$$

**Question No: 31 ( Marks: 10 )**

Use  $x_k^*$  as the left end point of each subinterval to find the area under  $y = mx$  over the interval  $[a, b]$ , where  $m > 0$  and  $a \geq 0$

Solution on next page

Suppose  $a = 1$   $b = 2$  so  $[a, b] = [1, 2]$

$x_k^* = x_{k-1} = a + (k-1)\Delta x$  (formula for left end point)

$$\Delta x = \frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$$

Suppose  $k$ th has area

$$f(x_{k^*})\Delta x = x_{k^*}\Delta x$$

$$\left[1 + \frac{k}{n}\right] \Delta x$$

$$\left[1 + \frac{k}{n}\right] \frac{1}{n}$$

$$\sum_{k=1}^n f(x_{k^*})\Delta x = \sum_{k=1}^n \left[1 + \frac{k}{n}\right] \frac{1}{n}$$

Area by solving

$$\begin{aligned} A &= \lim_{\Delta x \rightarrow 0} \sum_{k=1}^n f(x_{k^*})\Delta x = \lim_{\Delta x \rightarrow 0} \left[ \frac{3}{2} - 1 + \frac{1}{2n} \right] \\ &= \frac{3}{2} - 1 + 0 \end{aligned}$$

This paper is solved by our best knowledge. In the case of any error/correction/suggestion, please contact at [gulshanvu@yahoo.com](mailto:gulshanvu@yahoo.com), with reference to the concerned paper's number.

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