FINALTERM EXAMINATION
Fall 2009

Calculus & Analytical Geometry-I

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UPDATED VERSION
Exclusive thanks to Mahar Azahar (Lodhran)

Question No: 1   (Marks: 1)   - Please choose one

Let $y = \dfrac{x^2}{2}$. Find average rate of change of y with respect to x over the interval [3, 4]

► $\dfrac{25}{2}$

► $\dfrac{7}{2}$

► $\dfrac{25}{14}$

► $\dfrac{7}{14}$

►

Question No: 2   (Marks: 1)   - Please choose one

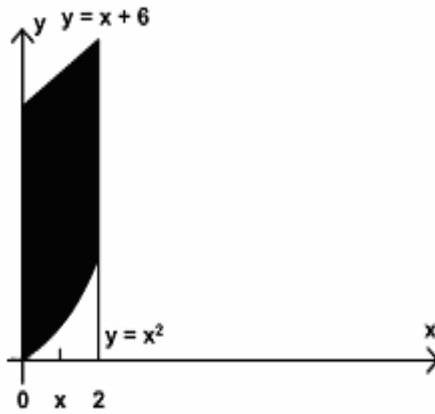If $2x - y = -3$ then $\dfrac{dy}{dx} =$

► **2**
► -2
► 0
► -3

Question No: 3   (Marks: 1)   - Please choose one
In the following figure, the area bounded on the sides by the lines are **:**

- ► $x = 0$
  - ► $x = 2$
  - ► $x = 0 \text{ and } x = 2$
  - ► $x = 6$

Question No: 4 (Marks: 1) - Please choose one

What is the sum of following series?
$1 + 2 + 3 + 4 + _____ + n$

- ► $\dfrac{n+1}{2}$

- ► $\dfrac{(n+1)(n+2)}{2}$

- ► $\dfrac{n(n+2)}{2}$

- ► $\dfrac{n(n+1)}{2}$

Question No: 5 (Marks: 1) - Please choose one

Let f is a smooth function on [0, 3]. What will be the arc length L of the curve y = f(x) from
x = 0 to x = 3?

$$L = \int_{0}^{3} \sqrt{1 + [f(x)]^2}\, dy$$

- ►

$$L = \int_a^b \sqrt{1+[f'(x)]^2}$$

►

$$L = \int_0^3 \sqrt{1+[f'(x)]^2}\,dy$$

►

$$L = \int_0^3 \sqrt{1+[f'(x)]^2}\,dx$$

►

Question No: 6   (Marks: 1)   - Please choose one

The PYTHAGORAS theorem describes the relationship between the sides of
………► **Right angle triangle** …………

- ► **Right angle triangle**
- ► Isoceleous triangle
- ► Equilateral triangle

Question No: 7   (Marks: 1)   - Please choose one

Which operation can not be applied on the functions?

- ► Subtraction
- ► **Cross product**
- ► Addition
- ► Composition

Question No: 8   (Marks: 1)   - Please choose one

The graph of the equation $y = x^2 - 4x + 5$ will represent

- ► **Parabola**
- ► Straight line
- ► Two straight lines
- ► Ellipse

Question No: 9   (Marks: 1)   - Please choose one

Polynomials are always …………………. functions

- ► **Continuous**
- ► Discontinuous

Question No: 10   (Marks: 1)   - Please choose one

The tan(x) is discontinuous at the points where

► Cos(x) =0
► Sin(x) =0
► **Tan(x) =0**

Question No: 11   (Marks: 1)   - Please choose one

A differentiable function must be differentiable on the interval

$(-\infty, \infty)$

► $(0, \infty)$

► $(-\infty, \infty)$

► (a, ∞) *where a is any negative* int *eger*

Question No: 12   (Marks: 1)   - Please choose one

Let $y = (x^3 + 2x)^{37}$. Which of the following is correct?

► $\dfrac{dy}{dx} = (37)(x^3 + 2x)^{36}$

► $\dfrac{dy}{dx} = 111x^2(x^3 + 2x)^{36}$

► $\dfrac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{36}$

► $\dfrac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{38}$

Question No: 13   (Marks: 1)   - Please choose one

Consider the indefinite integral $\displaystyle\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3}\, dx$

Let $t = x^3 + 2x^2 + x - 3$

Is the following substitution correct?

$\displaystyle\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3}\, dx = \int \frac{1}{t}\, dt$

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► Yes
► **No**

Question No: 14   (Marks: 1)   - Please choose one

$\log_b ac = $ _____

► $\log_b a + \log_b c$

► $\log_b a - \log_b c$

► $\dfrac{\log_b a}{\log_b c}$

► $(\log_b a)(\log_b c)$

Question No: 15   (Marks: 1)   - Please choose one

If a function has an extreme value (either a maximum or a minimum) on an open interval (a,b), then the extreme value occurs at ………….. of f

► First point
► Mid point
► **Critical point**
► End point

Question No: 16   (Marks: 1)   - Please choose one

The Mean Value Theorem states that "Let function f be differentiable on (a,b) and continuous on [a, b], then there exist at least one point c in (a,b) where   ……………… "

► $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

► $f(c) = \dfrac{f(b) - f(a)}{b - a}$

► $f(c) = \dfrac{f(a) - f(b)}{b - a}$

► $f'(c) = \dfrac{f(a) - f(b)}{b - a}$

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Question No: 17   (Marks: 1)   - Please choose one

$$\frac{d}{dx}[F(x)] = f(x)$$

If there is some function F such that   then any function of the form $F(x) + C$ is ------------------- of $f(x)$

► Derivative
► **Antiderivative**
► Slope
► Maximum value

Question No: 18   (Marks: 1)   - Please choose one

$$\sum_{k=1}^{n} f(x^{*}_{k}) \Delta x_{k}$$

The sum   is known as:

► **Riemann Sum**
► General Sum
► Integral Sum
► Geometric Sum

Question No: 19   (Marks: 1)   - Please choose one

$$\int_{0}^{\frac{\pi}{2}} \cos u \, du$$

If   , then which of the following is true?

► 2
► **1**
► 0
► -1

Question No: 20   (Marks: 1)   - Please choose one

$$\int_{0}^{\pi} \sin u \, du$$

If   , then which of the following is true?

► 1
► 2
► **0**
► -1

Question No: 21   (Marks: 1)   - Please choose one

$$\frac{d}{dx}[F(x)] = f(x)$$

If there is some function F such that   then antiderivatives
of $f(x)$ are $F(x) + C$. What does C represents?

► Polynomial
► **Constant**
► Dependent Variable
► Independent Variable

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Question No: 22   (Marks: 1)   - Please choose one

If f and g are continues function on an interval [a, b]

and $f(x) \geq g(x) \ \ for \ \ a \leq x \leq b$, then area is bounded by the lines parallel to:

► X -axis
► Y-axis
► **Both X -axis and Y-axis**
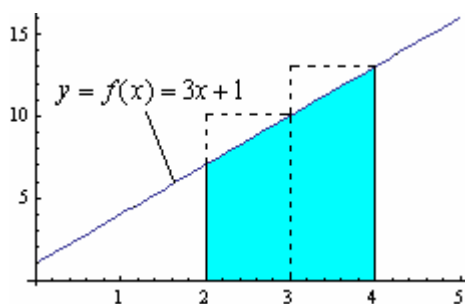
Question No: 23   (Marks: 1)   - Please choose one

$$\int_{1}^{2/3} dx = \underline{\hspace{2cm}}$$

► $\dfrac{-1}{3}$
► 0
► $\dfrac{1}{3}$
► $\dfrac{2}{3}$

Question No: 24   (Marks: 1)   - Please choose one

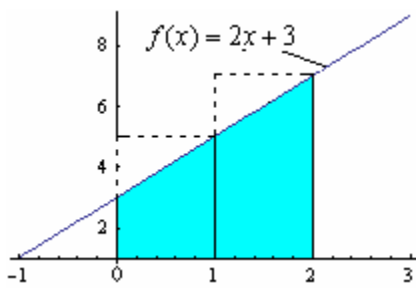$$\int_{0}^{2} x \ dx = \underline{\hspace{2cm}}$$

► **2**
► 0
► 2
► -2
► $\dfrac{x^2}{2}$

Question No: 25   (Marks: 1)   - Please choose one

Which of the following is approximate area of the shaded region by taking x_1^{*} and x_2^{*} as left endpoint of equal-length subintervals?

$y = f(x) = 3x + 1$

► 17
► **20**
► **23**
► 26

Question No: 26   (Marks: 1)   - Please choose one

Which of the following is approximate area of the shaded region by taking x_1^* and x_2^* as right endpoint of equal-length subintervals?

► 8
► **10**
► 12
► 14

Question No: 27   (Marks: 1)   - Please choose one

What is the length of each sub-interval, if the interval [1,3] is divided into n sub-intervals of equal length?

► $\dfrac{1}{n}$

► **$\dfrac{2}{n}$**

► $\dfrac{3}{n}$

► $\dfrac{4}{n}$

Question No: 28   (Marks: 1)   - Please choose one

Evaluate

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = -----$$

► **4**
► 2
► 1
► ∞

Question No: 29   (Marks: 1)   - Please choose one

$$\left\{ \frac{1}{2^n} \right\}_1^n$$

represents the sequence**:**

► $\dfrac{-1}{2}, \dfrac{-1}{4}, \dfrac{-1}{8}, \ldots$

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, ...$$

►

$$0, 1, \frac{1}{2}, \frac{1}{4}, ...$$

►

$$0, 1, 2, 3...$$

►

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Question No: 30   (Marks: 1)   - Please choose one

For a sequence $\{a_n\}$ if the difference between successive terms $a_{n+1} - a_n \leq 0$ then the sequence is known as**:**

- ► Increasing
- ► Decreasing
- ► Nondecreasing
- ► **Nonincreasing**

Question No: 31   (Marks: 1)   - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\dfrac{a_{n+1}}{a_n} > 1$ then the sequence is known as**:**

- ► **Increasing**
- ► Decreasing
- ► Nondecreasing
- ► Nonincreasing

Question No: 32   (Marks: 1)   - Please choose one

If the partial sum of a series is finite then the series will/will be**:**
► Divergent
- ► **Convergent**
- ► Give no information

Question No: 33   (Marks: 1)   - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + ... + ar^{k-1} + ... where (a \neq 0)$, $|r| < 1$ then which of the following is true for the given series?

- ► **Converges**
- ► Diverges
- ► Gives no information

Question No: 34   (Marks: 1)   - Please choose one

If $\rho = \lim\limits_{k \to +\infty} \sqrt[k]{u_k}$ where $\rho > 1$ then the series $\sum u_k$ with positive terms will /will be...........?

- ► **Convergent**

► Divergent
► Give no information

Question No: 35   (Marks: 1)   - Please choose one

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k}$$

Which of the following is true for the series   ?

► Arithmetic Series
► Geometric Series
► **Alternating Harmonic Series**
► Harmonic Series

Question No: 36   (Marks: 1)   - Please choose one

.................is the special case of Tylor's theorem.

► **Roll's Theorem**
► Picard's Method
► Integration
► Maclaurin's Theorem

Question No: 37   (Marks: 1)   - Please choose one

If f is integrable on a closed interval containing the four points *a*, *b*, *c* and *d* then

$$\int_{a}^{d} f(x)dx = \underline{\hspace{3cm}}$$

►
$$\int_{a}^{b} f(x)\,dx + \int_{b}^{c} f(x)\,dx + \int_{c}^{d} f(x)\,dx$$

►
$$\int_{a}^{b} f(x)\,dx + \int_{c}^{d} f(x)\,dx$$

►
$$\int_{a}^{c} f(x)\,dx + \int_{b}^{d} f(x)\,dx$$

►
$$\int_{a}^{d} f(x)\,dx$$

►

Question No: 38   (Marks: 1)   - Please choose one

Suppose f and g are integrable functions on [*a*, *b*] and *c* is a constant, then

$$\int_{a}^{b} c\,[f(x)+g(x)]\,dx = \underline{\hspace{3cm}}$$

$$\int_a^b f(cx)dx + \int_a^b g(cx)dx$$

►

$$\int_a^b f(x)\ dx + \int_a^b g(x)]\ dx$$

►

$$c\int_a^b f(x)dx + c\int_a^b g(x)dx$$

►

► 0

Question No: 39   (Marks: 1)   - Please choose one

What is the difference between the values of the

$$\int_a^b f(x)dx \quad and \quad \int_a^b f(t)dt$$

integrals   ?

► Differ by b-a
► Differ by a-b
► **No difference**
► Differ by b+a

Question No: 40   (Marks: 1)   - Please choose one

$$\int_{-1}^{2} f(x)\ dx = 5 \qquad \int_{-1}^{2} g(x)\ dx = -3$$

If   and   then which of the following is value

$$\int_{-1}^{2} [f(x) + 2g(x)]\ dx$$

of   ?

► **-1**
► -8
► 2
► 11

Question No: 41   (Marks: 2)

$$\frac{1}{1} + \frac{1}{8} + \frac{1}{27} + \dots\dots + \frac{1}{1000}$$

Express the sum   in sigma notation.

$$\sum_{n=1}^{10} (1/n^3)$$

Question No: 42   (Marks: 2)

Only write down the Maclaurin series for e^x

Question No: 43   (Marks: 2)

Evaluate the following integral:

$$\int_{1}^{4} \sqrt{x}\ dx$$

$$\int_{1}^{4} \sqrt{x}\ dx$$

$$= \int_{1}^{4} \sqrt{x}.1\,dx$$

$$= x\sqrt{x} + \int_{1}^{4} 1/\sqrt{x}.1\,dx$$

Question No: 44   (Marks: 3)

Evaluate the following sum:

$$\sum_{k=1}^{6} (k^2 - 5)$$
$$= -4 - 1 + 4 + 11 + 20 + 31 = 61$$

Question No: 45   (Marks: 3)

Find a definite integral indicating the area enclosed by the curves $y = x^2$, $x > 0$ and bounded on the sides by the lines $y = 1$ and $y = 4$. But do not evaluate the integral.

Question No: 46   (Marks: 3)

$$a_n = \left\{ \frac{3}{n^2} \right\}_{n=5}^{\infty}$$

Determine whether the following sequence is strictly monotone or not. If your answer is yes or no, then give reason .

Yes the sequence is strictly monotone because the denominator is increasing

Question No: 47   (Marks: 5)

The region bounded by the y-axis, the graph of the equation $x = y^{\frac{3}{2}}$ and the line $y = 2$ is revolved about y-axis. Find the volume of the resulting solid.

Question No: 48   (Marks: 5)

Compute the following sum:

$$\sum_{i=1}^{n} \left(4i^2 - i \right) = \left(4(1)^2 - 1 \right) + \left(4(2)^2 - 2 \right) + \left(4(3)^2 - 3 \right) + \left(4(4)^2 - 4 \right)..............$$
$$= 3 + 14 + 33 + 60......................$$

Question No: 49   (Marks: 5)

Use L'Hopital's rule to evaluate the limit

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos 2x}$$

$$\lim_{x \to \frac{\pi}{2}} (1 - \sin x) = 0 \quad \lim_{x \to \frac{\pi}{2}} (1 + \cos 2x) = 0$$

$$= 0/0$$

So by L'Hopital's rule

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos 2x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{d/dx(1 - \sin x)}{d/dx(1 + \cos 2x)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-2\sin 2x} = \frac{\cos \frac{\pi}{2}}{2 \sin \pi} = 0$$

Question No: 50   (Marks: 10)

$$\sum_{1}^{\infty} \frac{2^n}{n(n+2)}$$

Use the Ratio test to determine whether the series   converges or diverges.

$$p = \lim_{x \to \infty} \frac{u_{k+1}}{u_k} =$$

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