FINALTERM EXAMINATION
Fall 2008
(Session - 1)

Calculus & Analytical Geometry-I

Question No: 1 (Marks: 1) - Please choose one

If $y = f(x)$ then the average rate of change of y with respect to x over the interval $[x_0, x_1]$ is the .... Joining the points $(x_0, f(x_0)) and (x_1, f(x_1))$ on the graph of f

- ► Slope of the secant line
- ► Slope of tangent line
- ► Secant line
- ► none of these

Question No: 2 (Marks: 1) - Please choose one

Natural domain of $\frac{(x^2-4)}{(x-2)}$ is

- ► $(-\infty, 2)U(2, +\infty)$
- ► $(-\infty, 2)$
- ► $(-\infty, 0)$
- ► None of these

Question No: 3 (Marks: 1) - Please choose one

The equation $(x+4)^2+(y-1)^2=6$ represents a circle having center at ……….. and radius ....

- ► $(-4,1), \sqrt{6}$
- ► $(-4,1), 6$
- ► $(-4,-1), \sqrt{6}$
- ► None of these

Question No: 4 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to\infty} \sqrt[k]{u_k} = \lim_{k\to\infty} (u_k)^{\frac{1}{k}}$ if $\rho > 1$ then the series

- ► Converges
- ► **Diverges**
- ► May converge or diverge
- ► None of these

Question No: 5 (Marks: 1) - Please choose one

The series $\sum u_k \; and \; \sum v_k$ are convergent series then $(\sum u_k + \sum v_k) \; and \; (\sum u_k - \sum v_k)$ will be ……………and…………….

- ► Convergent, convergent
- ► Divergent, divergent
- ► **Convergent, divergent**
- ► Divergent, convergent

Question No: 6 (Marks: 1) - Please choose one

The notation $\{\frac{1}{2^n}\}_{-1}^{n}$ represents the sequence

- $2,1,\frac{1}{2},\frac{1}{4},\ldots$
- $0,1,2,3\ldots$
- $0,1,\frac{1}{2},\frac{1}{4},\ldots$
- None of these

Question No: 7   (Marks: 1)   - Please choose one

If f is continuous on (a,b] but does not have a limit from the right then the integral defined by $\int_a^b f(x)dx = \lim_{l \to a} \int_l^b f(x)dx$ is called .... Integral

- Improper
- Proper
- None of these

Question No: 8   (Marks: 1)   - Please choose one

An object is displaced 1m by a force of 1N then the work done W is

- 2
- 0
- None of these
- **1**

Question No: 9   (Marks: 1)   - Please choose one

If f is a smooth function on [a,b] then the arc length L of the curve y=f(x) from x=a to x=b will be

- $L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$

$$L = \int_a^b \sqrt{1+[f'(x)]}dx$$

▶

$$L = \int_0^a \sqrt{1+[f'(x)]}dy$$

▶

▶ None of these

Question No: 10 (Marks: 1) - Please choose one

If f is a smooth function on [0,3] then the arc length L of the curve y=f(x) from x=0 to x=3 will be

$$L = \int_0^3 \sqrt{1+[f'(x)]^2}dx$$

▶

$$L = \int_a^b \sqrt{1+[f'(x)]^2}$$

▶

$$L = \int_0^3 \sqrt{1+[f'(x)]^2}dy$$

▶

▶ None of these

Question No: 11 (Marks: 1) - Please choose one

By using cylindrical shell to find volume of the solid when the region R in the first quadrant enclosed between $y = 3x \text{ and } y = 2x^2$ is revolved about the x-axis

$$V = \int_0^{\frac{3}{2}} 2\pi x(3x - 2x^2)dx$$

▶

$$V = \int_0^{\frac{3}{2}} x(3x - 2x^2)dx$$

►

$$V = \int_0^{\frac{3}{2}} 2\pi(3x - 2x^2)dx$$

►

► None of these

Question No: 12 (Marks: 1) - Please choose one

By using cylindrical shell to find volume of the solid when the region R in the first quadrant enclosed between $y = x \text{ and } y = x^2$ is revolved about the y-axis is represented by

$$V = \int_0^{3} 2\pi x(x - x^2)dx$$

►

$$V = \int_0^{1} x(x - x^2)dx$$

►

$$V = \int_0^{1} 2\pi(x - x^2)dx$$

►

► None of these

Question No: 13 (Marks: 1) - Please choose one

If

$$\int_a^a f(x)dx =$$

a is in the domain of f, then

► $f'(x)$

► $f(x)$

► **0**
► None of these

Question No: 14 (Marks: 1) - Please choose one

Consider the integral $\int_0^2 x^2 dx$, the area on right is bounded by

► $y = x^2$
► $x = 2$
► $x = 0$
► None of these

Question No: 15 (Marks: 1) - Please choose one

The series $1-3+5-7+9-11$ may written as in sigma notation

► $\sum_{k=0}^{k=5} (-1)^k (2k+1)$
► $\sum_{k=1}^{k=5} (-1)^k (2k+1)$
► $\sum_{k=1}^{k=5} (2k+1)$
► None of these

Question No: 16 (Marks: 1) - Please choose one

$4^2+5^2+6^2+7^2$ in sigma notation may be represented as

$$\sum_{k=2}^{k=7} k^2$$

► $\sum_{k=2}^{k=7} (k+1)^2$

► $\sum_{k=4}^{k=7} k^2$

►

► None of these

Question No: 17 (Marks: 1) - Please choose one

If a function f is ……….. on a closed interval [a,b] ,then f has both a maximum and minimum value on [a,b]

► **Continuous**
► Discontinuous
► Differentiable
► None of these

Question No: 18 (Marks: 1) - Please choose one

Let f be a function on an interval, and x_1 and x_2 denote the points in that interval, if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ then the we can say that f is

► Increasing function
► **Decreasing function**
► Constant function
► None of these

Question No: 19 (Marks: 1) - Please choose one

If a function satisfies the conditions

f(c) is defined

$$\lim_{x \to c^+} f(x)$$

Exists

$$\lim_{x \to c^+} f(x) = f(c)$$

Then the function is said to be

► **Continuous at c**
► Continuous from left at c
► Continuous from right at c
► None of these

Question No: 20 (Marks: 1) - Please choose one

For a function f(x) to be continuous on interval (a,b) the function must be continuous

► **At all point in (a,b)**
► Only at a and b
► At mid point of a and b
► None of these

Question No: 21 (Marks: 2)

$$a_{n+1} = \frac{1}{3}(a_n + \frac{1}{a_n}) \ for\ n \geq 1\ and\ a_1 = 2$$

Write down the first two term of the sequence

Question No: 22 (Marks: 2)

Find the integral of the surface area of the portion of the sphere generated by revolving the

$$y = \sqrt{2 - x^2}\ , 0 \leq x \leq \frac{1}{3}$$

curve
(Note: Just find the integral do not solve the integral)

Question No: 23 (Marks: 2)

Calculate $\int_2^5 f(x)dx$ if $\int_2^3 f(x)dx = 7, \int_3^4 f(x)dx = 2, \int_5^4 f(x)dx = 5$

$$\int_2^3 f(x)dx = 7, \int_3^4 f(x)dx = 2, \int_5^4 f(x)dx = 5$$

$$\int_2^5 f(x)dx = 7 + 2 - 5 = 4$$

Question No: 24 (Marks: 3)

Find the first two Taylor polynomials for $\ln x$ about $x = 3$

Question No: 25 (Marks: 3)

Let the curve $y = x^{\frac{3}{2}}$; $0 \le y \le 2$, then find the surface area generated by revolving the curve. (But do not evaluate)

Question No: 26 (Marks: 3)

Express the sum $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + ... + \frac{1}{7225}$ in sigma notation but do not evaluate.

$$\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + ... + \frac{1}{7225}$$

$$\sum_{k=1}^{7225} k^3 + 1$$

Question No: 27 (Marks: 5)

Find the first four nonzero terms of the Taylor series generated by f at $x = a$

$$f(x) = \frac{1}{1-x} \quad at \quad x = 2$$

Question No: 28 (Marks: 5)

Evaluate the Definite Integral using the First fundamental Theorem of Calculus

$$\int_0^1 (x^5 - x^3 + 2x)dx$$

$$Let\, u = (x^5 - x^3 + 2x)$$

$$\int_0^1 (u)dx$$

Question No: 29   (Marks: 5)

Express the definite integrals as limits (Do not evaluate the integrals)

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (1 + Cosx)\, dx$$

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (1 + Cosx)\, dx$$

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \lim_{\max xk \to 0} \sum_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (1 + \cos x)dx$$

Question No: 30   (Marks: 10)

Find the region enclosed by the curves and also find the area

$$y = x^2 \;,\; y = \sqrt{x} \;,\; x = \frac{1}{4} \;, \quad x = 1$$

Question No: 31 (Marks: 10)

Use x^*_k as the left end point of each subinterval to find the area under

$y = mx \;\; over\, the\, \text{int}\, erval\left[a,b\right], where\, m > 0\, and\; a \ge 0$

Solution on next page

$Suppose \quad a = 1\; b = 2\; so[a,b] = [1,2]$

$x_k * = x_{k-1} = a + (k-1)\Delta x \qquad (formula\; for\, left\, end\;\, po\,\text{int})$

$\Delta x = \frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$

$Suppose\;\; kth\;\, has\;\, area$

$f\,(x_{k*)}\Delta x = x_{k*}\Delta x$

$\left[1+\frac{k}{n}\right]\Delta x$

$\left[1+\frac{k}{n}\right]\frac{1}{n}$

$\sum_{k=1}^{n} f\,(x_{k*})\Delta x = \sum_{k=1}^{n}[1+k-1]$

$Area\, by\, solving$

$$A = \lim_{\delta x \to 0}\sum_{k=1}^{n} f\,(x_{k*})\Delta x = \lim_{\delta x \to 0}\left[\frac{3}{2} - 1 + \frac{1}{2n}\right]$$

$$= \frac{3}{2} - 1 + 0$$