FINALTERM EXAMINATION 2009
(Session - 2)

Calculus & Analytical Geometry-I

Question No: 1 (Marks: 1) - Please choose one

According to Power-Rule of differentiation, if $f(x) = x^n$ where n is a real number, then

$\frac{d}{dx}[x^n] =$

- ▶ x^{n-1}
- ▶ $n\,x^{n-1}$
- ▶ $n\,x^{n+1}$
- ▶ $(n-1)x^{n+1}$

Question No: 2 (Marks: 1) - Please choose one

If a function g is differentiable at a point x and a function f is differentiable at a point g(x), then the _____ is differentiable at point x .

- ▶ Composition (f o g)
- ▶ Quotient (f / g)
- ▶ Product (f . g)
- ▶ Sum (f + g)

Question No: 3 (Marks: 1) - Please choose one

$y = f\left(g\left(h(x)\right)\right)$

If

$u = g\left(h(x)\right)$

$v = h(x)$ $\frac{dy}{dx} =$ _____

then

- ▶ $\frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$
- ▶ $\frac{dy}{du}\frac{du}{dv}\frac{dv}{dx}$
- ▶ $\frac{dv}{du} \cdot \frac{du}{dv} \cdot \frac{dy}{dx}$

Question No: 4 (Marks: 1) - Please choose one

If a function f is ..... on a closed interval [a,b] , then f has both maximum and minimum value on [a,b].

- ▶ Continuous
- ▶ Discontinuous
- ▶ None of these

Question No: 5 (Marks: 1) - Please choose one

$$\int_a^x \frac{t^2}{2} dt$$

The expression , represents a function of :

- ▶ t
- ▶ a
- ▶ Both x and a
- ▶ x

Question No: 6 (Marks: 1) - Please choose one

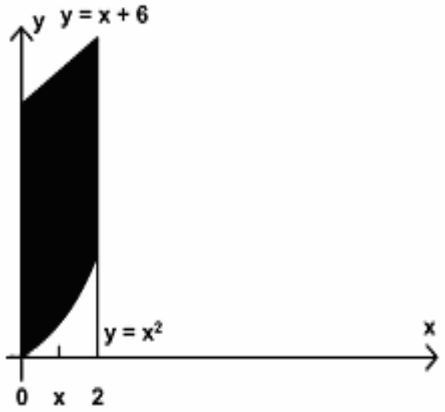
$\int cf(x)dx =$ _____

if c is a constant

- 0
- c
- $\int f(cx)dx$
- $c\int f(x)dx$

Question No: 7 (Marks: 1) - Please choose one

In the following figure, the area enclosed is bounded below by **:**



- $y = x + 6$
- $y = x^2$
- $x = 2$
- $x = 0$

Question No: 8 (Marks: 1) - Please choose one

At what points the two curves: $y = x^2 \ and \ y = x + 6$ intersect ?

- $x = 0 \ and \ x = 2$
- $x = 0 \ and \ x = 3$
- $x = 2 \ and \ x = 3$
- $x = -2 \ and \ x = 3$

Question No: 9 (Marks: 1) - Please choose one

What is the sum of following series?

$1+2+3+4+\_\_\_\_\_+n$

- $\frac{n+1}{2}$
- $\frac{(n+1)(n+2)}{2}$
- $\frac{n(n+2)}{2}$
- $\frac{n(n+1)}{2}$

Question No: 10 (Marks: 1) - Please choose one

$If\ b>0\ then\ \frac{d}{dx}[b^x] = \_\_\_\_\_$

- 0
- xb^{x-1}
- $\ln b$
- $b^x \ln b$

Question No: 11 (Marks: 1) - Please choose one

Let S be a solid bounded by two parallel planes perpendicular to the x-axis at x = a and x = b .If , for each x in [a,b], the cross-section area of S perpendicular to the x-axis is A(x) ,then what is the volume of the solid?

- $V=\int_a^b A(y)dx$
- $V=\int_a^b A(x)dx$

$$V = \int_{0}^{A(x)} [b-a]dx$$

▶

$$V = \int_{0}^{A(x)} [b+a]dx$$

▶

Question No: 12 (Marks: 1) - Please choose one

Let the solid generated by the region enclosed between $y = \sqrt{x}$; $x = 1, x = 4$ and the x-axis is revolved about the y-axis. Which of the following equation gives the volumes of a solid by cylindrical shells?

$$V = \int_{1}^{4} 2\pi x\sqrt{x}dx$$

▶

$$V = \int_{1}^{4} 2x\sqrt{x}dx$$

▶

$$V = \int_{0}^{4} 2x\sqrt{x}dx$$

▶

$$V = \int_{-4}^{4} 2x\sqrt{x}dx$$

▶

Question No: 13 (Marks: 1) - Please choose one

If slope m of a none vertical line is m =1 then the angle of inclination of the line is

$$\frac{\pi}{4}$$

▶

► $\frac{\pi}{2}$

► $\frac{\pi}{5}$

Question No: 14 (Marks: 1) - Please choose one

The PYTHAGORAS theorem describes the relationship between the sides of .....

► Right angle triangle
► Isoceleous triangle
► Equilateral triangle

Question No: 15 (Marks: 1) - Please choose one

If a quantity y depends on another quantity x in such a way that each value of x determines exactly one value of y , we say that y is ..... of x

► relation
► function
► not a function
► not a relation

Question No: 16 (Marks: 1) - Please choose one

The graph of the equation $y = x^2 - 4x + 5$ will represent

► Parabola
► Straight line
► Two straight lines

► Ellipse

Question No: 17 (Marks: 1) - Please choose one

The $\lim_{x \to a} f(x) = ...where\ f(x) = k$ (k is a constant)

► k+2
► k+1
► k

Question No: 18 (Marks: 1) - Please choose one

$$\int \frac{1}{x^2+2}\, dx$$

Consider the indefinite integral

Let $t = x^2 + 2$

Is the following substitution correct?

$\int \frac{1}{x^2+2}\, dx = \int \frac{1}{t}\, dt$

► Yes
► No

Question No: 19 (Marks: 1) - Please choose one

$\log_b \frac{1}{t} =$ _____

► $\log_b t$
► $1 - \log_b t$
► $1 + \log_b t$
► $-\log_b t$

Question No: 20 (Marks: 1) - Please choose one

How the series $1-3+5-7+9-11$ can be expressed in sigma notation?

► $\sum_{k=0}^{k=5} (-1)^k (2k+1)$

$$\sum_{k=1}^{k=5}(-1)^k(2k+1)$$

►

$$\sum_{k=1}^{k=5}(2k+1)$$

►

$$\sum_{k=1}^{k=5}(2k+1)$$

►

Question No: 21 (Marks: 1) - Please choose one

$$\sum_{k=1}^{n} f(x^*_k)\Delta x_k$$

The sum is known as:

► Riemann Sum
► General Sum
► Integral Sum
► Geometric Sum

Question No: 22 (Marks: 1) - Please choose one

$$\sum_{k=1}^{n} f(x^*_k)\Delta x_k$$

What does 'n' represent in the Riemann Sum ?

► No. of Circles
► No. of Subintervals
► No. of Loops
► No. of Squares

Question No: 23 (Marks: 1) - Please choose one

If *w* and *v* are continuous functions of *y* on an interval [c, d] and $w(y) \geq v(y) \ for \ c \leq y \leq d$, then area is bounded by the lines parallel to: **:**

► Y-axis
► X-axis
► Both X-axis and Y-axis

Question No: 24 (Marks: 1) - Please choose one

How the series $2(1)+2(2)+2(3)+2(4)+2(5)$ can be expressed in sigma notation?

- $\sum_{k=0}^{5} 2k^2$
- $\sum_{k=1}^{5} 2k^2$
- $\sum_{k=0}^{5} 2k$
- $\sum_{k=1}^{5} 2k$

Question No: 25 (Marks: 1) - Please choose one

$\sum_{k=1}^{n} \frac{k^3}{2} =$ _____

- $\frac{n(n+1)}{4}$
- $\frac{[n(n+1)]^2}{8}$
- $\frac{n(n+1)(2n+1)}{12}$
- $\frac{(n+1)(2n+1)}{6}$

Question No: 26 (Marks: 1) - Please choose one

If $a_1 < a_2 < a_3 < ..... < a_n < ....$, then a sequence $\{a_n\}$ is.....

- Nondecreasing
- Decreasing

► Increasing
► Nonincreasing

Question No: 27 (Marks: 1) - Please choose one

If $a_1 \geq a_2 \geq a_3 \geq ..... \geq a_n \geq ....$, then a sequence $\{a_n\}$ is .....

► Increasing
► Nondecreasing
► Decreasing
► Nonincreasing

Question No: 28 (Marks: 1) - Please choose one

If the difference between successive terms $a_{n+1} - a_n > 0$ then the sequence $\{a_n\}$ is known as**:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 29 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} > 1$ then the sequence is known as**:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 30 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} \geq 1$ then the sequence is known as **:**

► Increasing
► Decreasing
► Nondecreasing

► Nonincreasing

Question No: 31 (Marks: 1) - Please choose one

If $f(n) = a_n$ is the nth term of the sequence and $f(n)$ is differentiable and $f'(n) > 0$ then the sequence will be**:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 32 (Marks: 1) - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + ... + ar^{k-1} + ... \ \ where\ (a \neq 0)$, $|r| < 1$ then which of the following is true for the given series?

► Converges
► Diverges
► Gives no information

Question No: 33 (Marks: 1) - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + ... + ar^{k-1} + ... \ \ where\ (a \neq 0)$, $|r| \geq 1$ then which of the following is true for the given series?

► Converges
► Diverges
► Gives no information

Question No: 34 (Marks: 1) - Please choose one

If the series $\sum_{k=1}^{\infty} u_k$ converges but the series $\sum_{k=1}^{\infty} |u_k|$ does not converge , then $\sum_{k=1}^{\infty} u_k$ will _____

► Converge absolutely
► Diverge
► Converge conditionally

Question No: 35 (Marks: 1) - Please choose one

Let $\sum u_k$ be a series with nonzero terms and suppose that

$$\rho = \lim_{k\to\infty} \frac{|u_{k+1}|}{|u_k|} > 1$$

then which of the following is true?

► The series $\sum |u_k|$ converges

► The series $\sum |u_k|$ diverges

► No conclusion can be drawn.

Question No: 36 (Marks: 1) - Please choose one

Suppose f and g are integrable functions on [*a*, *b*] and *c* is a constant, then

$\int_a^b c\,[f(x)+g(x)]\,dx =$ _____

► $\int_a^b f(cx)dx + \int_a^b g(cx)dx$

► $\int_a^b f(x)\,dx + \int_a^b g(x)]\,dx$

► $c\int_a^b f(x)dx + c\int_a^b g(x)dx$

►

► 0

Question No: 37 (Marks: 1) - Please choose one

Which of the following is surface area S generated by revolving the curve *y = f(x)* between *x = 0* and *x = 2* about the *x*-axis?

$$S = \int_0^2 2\pi f(x)\sqrt{1+[f(x)]}dx$$

▶

$$S = \int_0^2 2\pi f(x)\sqrt{1+[f'(x)]^2}dx$$

▶

$$S = \int_0^2 2\pi f(x)\sqrt{1+[f'(x)]}dx$$

▶

$$S = \int_0^2 2\sqrt{1+[f'(x)]}dx$$

▶

Question No: 38 (Marks: 1) - Please choose one

Which of the following is area of the surface generated by revolving the curve $y = 4\sqrt{x}\ ;\ 1 \le x \le 4$ about the *x*-axis?

$$\int_1^4 2\pi\left(4\sqrt{x}\right)\sqrt{1+\left[\left(4\sqrt{x}\right)\right]^2}\,dx$$

▶

$$\int_1^4 2\pi\left(4\sqrt{x}\right)\sqrt{1+\left[\left(4\sqrt{x}\right)'\right]}\,dx$$

▶

$$\int_1^4 2\pi+\sqrt{1+\left[\left(4\sqrt{x}\right)'\right]^2}\,dx$$

▶

$$\int_1^4 2\pi\left(4\sqrt{x}\right)\sqrt{1+\left[\left(4\sqrt{x}\right)'\right]^2}\,dx$$

▶

Question No: 39 (Marks: 1) - Please choose one

Which of the following is the work done *W* if an object moves in the positive direction along a coordinate line while subject to a force *F(x)* in the direction of motion over an interval [0,3]?

$$W = \int_{2}^{3} 3x dx$$

▶

$$W = \int_{0}^{3} 3x dx$$

▶

$$W = \int_{0}^{3} F(x) dx$$

▶

$$W = \int_{3}^{0} F(x) dx$$

▶

Question No: 40 (Marks: 1) - Please choose one

If $\int_{1}^{0} f(x)\, dx = 2$ and $\int_{0}^{5} f(x)\, dx = 1$ then which of the following is value of $\int_{1}^{5} f(x)\, dx$?

▶ -3
▶ -1
▶ 1
▶ 3

Question No: 41 (Marks: 2)

Derivative of a function is $\frac{1}{2\sqrt{x}}$. Find the original function.

Question No: 42 (Marks: 2)

If $\sum u_k$ is a series with positive terms and $\rho = \lim_{k \to +\infty} \sqrt[k]{u_k}$; then write the three cases for the series to be convergent, divergent or none.

Question No: 43 (Marks: 2)

Evaluate the following integral:

$$\int_1^3 \frac{1}{x^2}\,dx$$

Question No: 44 (Marks: 3)

Use the first fundamental theorem of calculus to evaluate the definite integral:

$$\int_0^2 f(x)\,dx \qquad where \qquad f(x)=\begin{cases} x^2 & ; \quad 0 \le x < 1 \\ x^3 & ; \quad 1 \le x \le 2 \end{cases}$$

Question No: 45 (Marks: 3)

$$\sum_{k=2}^{\infty}(-1)^{k-1}\frac{2^{k-1}}{(k-1)!}$$

Show that the series converges absolutely.

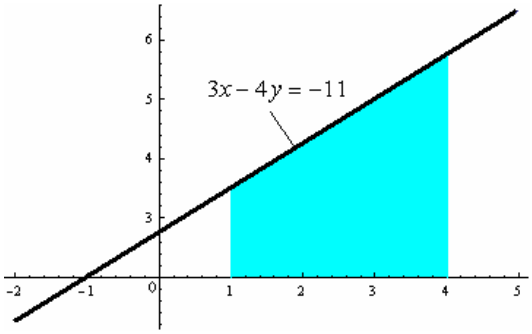
Question No: 46 (Marks: 3)

Express the following definite integral as limit of Riemann Sum. (Do not evaluate the integral)

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} Cos\,x\;dx$$

Question No: 47 (Marks: 5)

Express area of the shaded region as a definite integral.



Question No: 48 (Marks: 5)

How much work is required to wind the chain onto the pulley if a 100-ft length of steel chain weighting
15 lb/ ft. is dangling from a pulley?

Question No: 49 (Marks: 5)

Evaluate the following integral:

$$\int_1^2 \frac{x^2+\sqrt{x}}{x^2}\,dx$$

Question No: 50 (Marks: 10)

Use L'Hopital's Rule to evaluate

$$\lim_{x\to 0} (1+\sin x)^{\cot x}$$