FINALTERM EXAMINATION
Fall 2009
Calculus & Analytical Geometry-I

Question No: 1 (Marks: 1) - Please choose one

Let $y = \frac{x^2}{2}$. Find average rate of change of y with respect to x over the interval [3, 4]

- ► $\frac{25}{2}$
- ► $\frac{7}{2}$
- ► $\frac{25}{14}$
- ► $\frac{7}{14}$

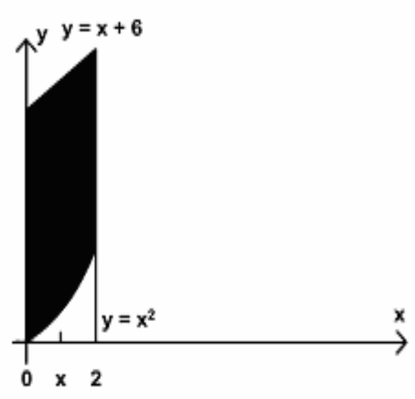
Question No: 2 (Marks: 1) - Please choose one

If $2x - y = -3$ then $\frac{dy}{dx} =$

- ► **2**
- ► -2
- ► 0
- ► -3

Question No: 3 (Marks: 1) - Please choose one

In the following figure, the area bounded on the sides by the lines are **:**



- $x = 0$
- $x = 2$
- $x = 0 \; and \; x = 2$
- $x = 6$

Question No: 4 (Marks: 1) - Please choose one

What is the sum of following series?

$1 + 2 + 3 + 4 + \_\_\_\_\_ + n$

- $\frac{n+1}{2}$
- $\frac{(n+1)(n+2)}{2}$
- $\frac{n(n+2)}{2}$
- $\frac{n(n+1)}{2}$

Question No: 5 (Marks: 1) - Please choose one

Let f is a smooth function on [0, 3]. What will be the arc length L of the curve y = f(x) from x = 0 to x = 3?

- $L = \int_0^3 \sqrt{1 + [f(x)]^2}\, dy$

$$L=\int_a^b \sqrt{1+[f'(x)]^2}$$

►

$$L=\int_0^3 \sqrt{1+[f'(x)]^2}\,dy$$

►

$$L=\int_0^3 \sqrt{1+[f'(x)]^2}\,dx$$

►

Question No: 6 (Marks: 1) - Please choose one

The PYTHAGORAS theorem describes the relationship between the sides of .....► **Right angle triangle** .....

► **Right angle triangle**
► Isoceleous triangle
► Equilateral triangle

Question No: 7 (Marks: 1) - Please choose one

Which operation can not be applied on the functions?

► Subtraction
► **Cross product**
► Addition
► Composition

Question No: 8 (Marks: 1) - Please choose one

The graph of the equation $y=x^2-4x+5$ will represent

► **Parabola**
► Straight line
► Two straight lines

► Ellipse

Question No: 9 (Marks: 1) - Please choose one

Polynomials are always ..... functions

► **Continuous**
► Discontinuous

Question No: 10 (Marks: 1) - Please choose one

The tan(x) is discontinuous at the points where

- ► Cos(x) =0
- ► Sin(x) =0
- ► **Tan(x) =0**

Question No: 11 (Marks: 1) - Please choose one

A differentiable function must be differentiable on the interval

$(-\infty, \infty)$

- ► $(0, \infty)$
- ► $(-\infty, \infty)$
- ► (a, ∞) *where a is any negative* int *eger*

Question No: 12 (Marks: 1) - Please choose one

Let $y = (x^3 + 2x)^{37}$. Which of the following is correct?

- $\frac{dy}{dx} = (37)(x^3 + 2x)^{36}$
- ► $\frac{dy}{dx} = 111x^2(x^3 + 2x)^{36}$
- ► $\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{36}$
- ► $\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{38}$

Question No: 13 (Marks: 1) - Please choose one

$$\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3}\, dx$$

Consider the indefinite integral

Let $t = x^3 + 2x^2 + x - 3$

Is the following substitution correct?

$$\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3}\, dx = \int \frac{1}{t}\, dt$$

- ► Yes
- ► **No**

Question No: 14 (Marks: 1) - Please choose one

$\log_b ac =$ _____

► $\log_b a + \log_b c$

► $\log_b a - \log_b c$

► $\dfrac{\log_b a}{\log_b c}$

► $(\log_b a)(\log_b c)$

Question No: 15 (Marks: 1) - Please choose one

If a function has an extreme value (either a maximum or a minimum) on an open interval (a,b), then the extreme value occurs at ..... of f

► First point

► Mid point

► **Critical point**

► End point

Question No: 16 (Marks: 1) - Please choose one

The Mean Value Theorem states that "Let function *f* be differentiable on (a,b) and continuous on [a, b], then there exist at least one point c in (a,b) where ..... "

► $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

► $f(c) = \dfrac{f(b) - f(a)}{b - a}$

► $f(c) = \dfrac{f(a) - f(b)}{b - a}$

► $f'(c) = \dfrac{f(a) - f(b)}{b - a}$

Question No: 17 (Marks: 1) - Please choose one

If there is some function *F* such that $\dfrac{d}{dx}[F(x)] = f(x)$ then any function of the form $F(x) + C$ is ---------- of *f*(*x*)

► Derivative
► **Antiderivative**
► Slope
► Maximum value

Question No: 18 (Marks: 1) - Please choose one

$$\sum_{k=1}^{n} f(x^*_k)\,\Delta x_k$$

The sum is known as:

► **Riemann Sum**
► General Sum
► Integral Sum
► Geometric Sum

Question No: 19 (Marks: 1) - Please choose one

$$\int_0^{\frac{\pi}{2}} \cos u\, du$$

If , then which of the following is true?

► 2
► **1**
► 0
► -1

Question No: 20 (Marks: 1) - Please choose one

$$\int_0^{\pi} \sin u\, du$$

If , then which of the following is true?

► 1
► 2
► **0**
► -1

Question No: 21 (Marks: 1) - Please choose one

$$\frac{d}{dx}[F(x)] = f(x)$$

If there is some function *F* such that then antiderivatives of $f(x)$ are $F(x)+C$. What does *C* represents?

► Polynomial
► **Constant**
► Dependent Variable
► Independent Variable

Question No: 22 (Marks: 1) - Please choose one

If *f* and *g* are continues function on an interval [a, b] and $f(x) \geq g(x) \ \ for \ \ a \leq x \leq b$, then area is bounded by the lines parallel to:

► X -axis
► Y-axis
► Both X -axis and Y-axis

Question No: 23 (Marks: 1) - Please choose one

$\int_{1}^{2/3} dx = \_\_\_\_\_$

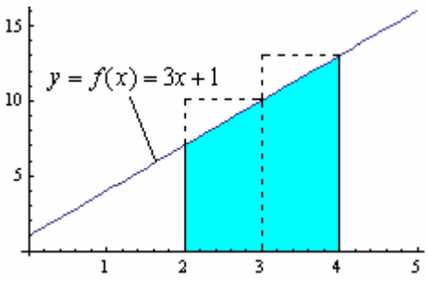
► $\frac{-1}{3}$
► 0
► $\frac{1}{3}$
► $\frac{2}{3}$

Question No: 24 (Marks: 1) - Please choose one

$\int_{0}^{2} x \ dx = \_\_\_\_\_$

► **2**

► 0
► **2**
► -2
► $\frac{x^2}{2}$

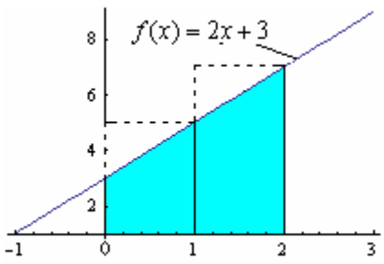


Question No: 25 (Marks: 1) - Please choose one

Which of the following is approximate area of the shaded region by taking x_1^* and x_2^* as left endpoint of equal-length subintervals?

► 17
► 20
► **23**
► 26

Question No: 26 (Marks: 1) - Please choose one



Which of the following is approximate area of the shaded region by taking x_1^* and x_2^* as right endpoint of equal-length subintervals?

► 8
► 10
► 12
► 14

Question No: 27 (Marks: 1) - Please choose one

What is the length of each sub-interval, if the interval [1,3] is divided into *n* sub-intervals of equal length?

► $\frac{1}{n}$
► $\frac{2}{n}$
► $\frac{3}{n}$
► $\frac{4}{n}$

Question No: 28 (Marks: 1) - Please choose one

Evaluate

$$\lim_{x\to 2}\frac{x^2-4}{x-2}=-----$$

► **4**
► 2
► 1
► ∞

Question No: 29 (Marks: 1) - Please choose one

$$\left\{\frac{1}{2^n}\right\}_1^n$$

represents the sequence**:**

$$\frac{-1}{2},\frac{-1}{4},\frac{-1}{8},\cdots$$

►

► $\frac{1}{2},\frac{1}{4},\frac{1}{8},...$

► $0,1,\frac{1}{2},\frac{1}{4},...$

► $0,1,2,3...$

Question No: 30 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the difference between successive terms $a_{n+1}-a_n \le 0$ then the sequence is known as**:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 31 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n}>1$ then the sequence is known as**:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 32 (Marks: 1) - Please choose one

If the partial sum of a series is finite then the series will/will be**:**

► Divergent
► Convergent
► Give no information

Question No: 33 (Marks: 1) - Please choose one

If the geometric series $a+ar+ar^2+ar^3+...+ar^{k-1}+...\ \ where\ (a\neq 0)$, $|r|<1$ then which of the following is true for the given series?

► Converges
► Diverges
► Gives no information

Question No: 34 (Marks: 1) - Please choose one

If $\rho=\lim_{k\to+\infty}\sqrt[k]{u_k}$ where $\rho>1$ then the series $\sum u_k$ with positive terms will /will be.....?

► Convergent

► Divergent
► Give no information

Question No: 35 (Marks: 1) - Please choose one

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k}$$

Which of the following is true for the series ?

► Arithmetic Series
► Geometric Series
► **Alternating Harmonic Series**
► Harmonic Series

Question No: 36 (Marks: 1) - Please choose one

.....is the special case of Tylor's theorem.

► **Roll's Theorem**
► Picard's Method
► Integration
► Maclaurin's Theorem

Question No: 37 (Marks: 1) - Please choose one

If f is integrable on a closed interval containing the four points *a*, *b*, *c* and *d* then

$$\int_a^d f(x)dx = \_\_\_\_\_$$

► $\int_a^b f(x)\,dx+\int_b^c f(x)\,dx+\int_c^d f(x)\,dx$

► $\int_a^b f(x)\,dx+\int_c^d f(x)\,dx$

► $\int_a^c f(x)\,dx+\int_b^d f(x)\,dx$

► $\int_a^d f(x)\,dx$

►

Question No: 38 (Marks: 1) - Please choose one

Suppose f and g are integrable functions on [*a*, *b*] and *c* is a constant, then

$$\int_a^b c\,[f(x)+g(x)]\,dx = \_\_\_\_\_$$

$\int_a^b f(cx)dx + \int_a^b g(cx)dx$

► $\int_a^b f(x)\ dx + \int_a^b g(x)]\ dx$

► $c\int_a^b f(x)dx + c\int_a^b g(x)dx$

►

► 0

Question No: 39 (Marks: 1) - Please choose one

What is the difference between the values of the $\int_a^b f(x)dx \quad and \quad \int_a^b f(t)dt$ integrals ?

► Differ by b-a
► Differ by a-b
► **No difference**
► Differ by b+a

Question No: 40 (Marks: 1) - Please choose one

If $\int_{-1}^{2} f(x)\ dx = 5$ and $\int_{-1}^{2} g(x)\ dx = -3$ then which of the following is value of $\int_{-1}^{2} [f(x) + 2g(x)]\ dx$?

► **-1**
► -8
► 2
► 11

Question No: 41 (Marks: 2)

Express the sum $\frac{1}{1} + \frac{1}{8} + \frac{1}{27} + ... + \frac{1}{1000}$ in sigma notation.

$\sum_{n=1}^{10} (1/n^3)$

Question No: 42 (Marks: 2)

Only write down the Maclaurin series for e^x

Question No: 43 (Marks: 2)

Evaluate the following integral:

$$\int_1^4 \sqrt{x}\ dx$$

$$\int_1^4 \sqrt{x}\, dx$$

$$= \int_1^4 \sqrt{x}.1\, dx$$

$$= x\sqrt{x} + \int_1^4 1/\sqrt{x}.1\, dx$$

Question No: 44 (Marks: 3)

Evaluate the following sum:

$$\sum_{k=1}^{6} (k^2 - 5)$$

$$= -4 - 1 + 4 + 11 + 20 + 31 = 61$$

Question No: 45 (Marks: 3)

Find a definite integral indicating the area enclosed by the curves $y = x^2 \ , \ x > 0$ and bounded on the sides by the lines $y = 1$ and $y = 4$. But do not evaluate the integral.

Question No: 46 (Marks: 3)

$$a_n = \left\{ \frac{3}{n^2} \right\}_{n=5}^{\infty}$$

Determine whether the following sequence is strictly monotone or not.If your answer is yes or no, then give reason .

Yes the sequence is strictly monotone because the denominator is increasing

Question No: 47 (Marks: 5)

The region bounded by the *y*-axis, the graph of the equation $x = y^{\frac{3}{2}}$ and the line *y* = 2 is revolved about *y*-axis. Find the volume of the resulting solid.

Question No: 48 (Marks: 5)

Compute the following sum:

$$\sum_{i=1}^{n} \left(4i^2 - i\right) = \left(4(1)^2 - 1\right) + \left(4(2)^2 - 2\right) + \left(4(3)^2 - 3\right) + \left(4(4)^2 - 4\right) \ldots\ldots\ldots\ldots$$

$$= 3 + 14 + 33 + 60 \ldots\ldots\ldots\ldots\ldots$$

Question No: 49 (Marks: 5)

Use L'Hopital's rule to evaluate the limit

$$\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{1+\cos 2x}$$

$$\lim_{x\to\frac{\pi}{2}} (1-\sin x)=0 \lim_{x\to\frac{\pi}{2}} (1+\cos 2x)=0$$

$$=0/0$$

So by L'Hopital's rule

$$\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{1+\cos 2x}$$

$$=\lim_{x\to\frac{\pi}{2}} \frac{d/dx(1-\sin x)}{d/dx(1+\cos 2x)}$$

$$=\lim_{x\to\frac{\pi}{2}} \frac{-\cos x}{-2\sin 2x}=\frac{\cos\frac{\pi}{2}}{2\sin\pi}=0$$

Question No: 50 (Marks: 10)

$$\sum_{1}^{\infty} \frac{2^n}{n(n+2)}$$

Use the Ratio test to determine whether the series converges or diverges.

$$p=\lim_{x\to\infty} \frac{u_{k+1}}{u_k}=$$