Practice Exercise lecture no. 4

Q1. Find the slopes of the sides of the triangle with vertices (-1, 3), (5, 4) and (2, 8).

Solution: Let A(-1,3), B(5,4) and C(2,8) be the given points, then

Slope of side AB = $\frac{4-3}{5+1}=\frac{1}{6}$

Slope of side BC = $\frac{8-4}{2-5}=\frac{-4}{3}$

Slope of side CA = $\frac{3-8}{-1-2}=\frac{5}{3}$

Q2. Find equation of the line passing through the point (1,2) and having slope 3.

Solution:
Point-slope form of the line passing through $P\left(x_1, y_1\right)$ and having slope m is given by the equation:

$y-y_1=m\left(x-x_1\right)$

$\Rightarrow \quad y\text{-}2=3\left(x\text{-}1\right)$

$\Rightarrow \quad y-2=3x-3$

$\Rightarrow \quad y=3x-1$

Q3. Find the slope-intercept form of the equation of the line that passes through the point (5,-3) and perpendicular to line $y=2x+1$.

Solution:
The slope-intercept form of the line with y-intercept b and slope m is given by the equation: $y=mx+b$

The given line has slope 2, so the line to be determined will have slope $m=-\frac{1}{2}$.

Substituting this slope and the given point in the point-slope form: $y-y_1=m\left(x-x_1\right)$, yields

$$y-(-3)=-\frac{1}{2}(x-5)$$

$$\Rightarrow y+3=-\frac{1}{2}(x-5)$$

$$\Rightarrow y = -\frac{1}{2}x + \frac{5}{2} - 3 \Rightarrow y = -\frac{1}{2}x - \frac{1}{2}$$

Q4. Find the slope and angle of inclination of the line joining the points (2, 3) and (-1, 2).

Solution: If m is the slope of line joining the points (2, 3) and (-1, 2) then

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{-1-2} = \frac{1}{3}$ is the slope

Now angle of inclination is:

$$\tan\theta = m$$

$$\tan\theta = \frac{1}{3}$$

$$\theta = \tan^{-1}(\frac{1}{3}) = 18.43^o$$

Q5. By means of slopes, Show that the points lie on the same line

A (-3, 4); B (3, 2); C (6, 1)

Solution: Slope of line through A(-3, 4) ; B(3, 2) $= \frac{2-4}{3+3} = -\frac{2}{6} = -\frac{1}{3}$

Slope of line through B(3, 2) ; C(6, 1) $= \frac{1-2}{6-3} = -\frac{1}{3}$

Slope of line through C(6, 1) ; A(-3, 4) $= \frac{4-1}{-3-6} = -\frac{3}{9} = -\frac{1}{3}$

Since all slopes are same, so the given points lie on the same line.

Practice Exercise
Lecture No.5: Distance, Circles, Equations

Solution 1:

The formula to find the distance between any two points (x_1, y_1) and (x_2, y_2) in the coordinate plane is given as

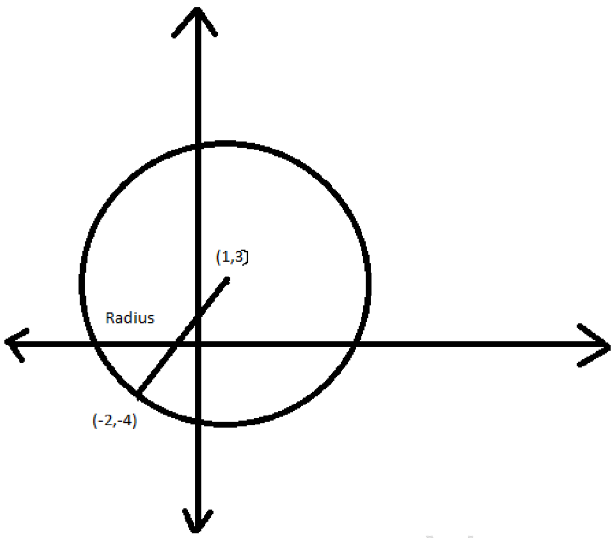
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The given points are $(5,6)$ and $(2,4)$, so the distance between these two points will be

$$\begin{aligned} d &= \sqrt{(2-5)^2+(4-6)^2} \\ &= \sqrt{(-3)^2+(-2)^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \end{aligned}$$

Solution 2:

It is given that center of the circle is $(1,3)$. We are also given a point on the circle that is $(-2,-4)$ as shown below.



The radius of the circle will be the distance between the points (1,3) and (-2,-4). That is

$$\begin{aligned} \text{Radius} = d &= \sqrt{[1-(-2)]^2+[3-(-4)^2} \\ &= \sqrt{(3)^2+(7)^2} \\ &= \sqrt{9+49} = \sqrt{58} \end{aligned}$$

Solution 3:

The general form of the equation of circle is given as

$$4x^2+4y^2-16x-24y+51=0$$

It can be re-written as

$$(4x^2-16x)+(4y^2-24y)=-51$$

$$(2x)^2-2(8x)+(2y)^2-2(12y)=-51$$

In order to complete the squares on the left hand side, we have to add 16 and 36 on both sides, it will then become

$$(2x)^2 - 2(8x) + 16 + (2y)^2 - 2(12y) + 36 = -51 + 16 + 36$$
$$(2x)^2 - 2(2x)(4) + (4^2) + (2y)^2 - 2(2y)(6) + (6)^2 = 1$$
$$(2x-4)^2 + (2y-6)^2 = 1$$
$$(x-2)^2 + (y-3)^2 = \left(\frac{1}{4}\right)$$

Comparing it with the standard form of the equation, the center of the circle will be $(2,3)$ and the radius will be $\frac{1}{2}$.

Solution 4:

The general form of the equation of circle is given as

$$2x^2 + 2y^2 + 6x - 8y + 12 = 0$$

It can be re-written as

$$(2x^2 + 6x) + (2y^2 - 8y) = -12$$
$$(x^2 + 3x) + (y^2 - 4y) = -6$$

In order to complete the squares on the left hand side, we have to add $\frac{9}{4}$ and 4 on both sides, it will then become

$$(x^2 + 3x + \frac{9}{4}) + (y^2 - 4y + 4) = -6 + \frac{9}{4} + 4$$
$$(x^2 + 2(x)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 + (y)^2 - 2(y)(2) + (2)^2 = \frac{1}{4}$$
$$\left(x + \frac{3}{2}\right)^2 + (y-2)^2 = \frac{1}{4}$$

Comparing it with the standard form of the equation, the center of the circle will be $\left(-\frac{3}{2}, 2\right)$ and radius will be $\frac{1}{2}$.

Solution 5:

The general form of the equation of circle is given as

$$x^2 + y^2 - 4x - 6y + 8 = 0$$

This can be re-written as

$$(x^2 - 4x) + (y^2 - 6y) = -8$$

In order to complete the squares on the left hand side, we have to add 4 and 9 on both sides, it will then become

$$(x^2-4x+4)+(y^2-6y+9)=-8+4+9$$
$$(x)^2-2(x)(2)+(2)^2+(y)^2-2(y)(3)+(3)^2=5$$
$$(x-2)^2+(y-3)^2=5$$

Comparing it with the standard form of the equation, the center of the circle will be (2,3) and the radius will be $\sqrt{5}$.

SOLUTION 6:

$$\because\ 3x^2+6x+3y^2+18y-6=0,$$
$$\Rightarrow 3(x^2+2x+y^2+6y-2)=0, \quad (\because \text{ taking 3 as common})$$
$$\Rightarrow x^2+2x+y^2+6y-2=0, \quad (\because \text{ dividing by 3 on both sides})$$
$$\Rightarrow x^2+2x+1+y^2+6y+9=2+9+1,$$
$$\Rightarrow (x+1)^2+(y+3)^2=12,$$
$$\Rightarrow (x+1)^2+(y+3)^2=(\sqrt{12})^2,$$
$$\Rightarrow (x-(-1))^2+(y-(-3))^2=(\sqrt{12})^2,$$
$\therefore$ Centre of the circle is (-1,-3) and radius is $\sqrt{12}$.

SOLUTION 7:

$$x^2-6x+y^2-8y=0, \quad (\because \text{ rearranging the term})$$
$$x^2-6x+y^2-8y+(3)^2=(3)^2, \quad (\because \text{ adding } (3)^2 \text{ on both sides})$$
$$(x^2-6x+9)+y^2-8y=9,$$
$$(x^2-6x+9)+y^2-8y+(4)^2=9+(4)^2, \quad (\because \text{ adding } (4)^2 \text{ on both sides})$$
$$(x^2-6x+9)+(y^2-8y+16)=9+16,$$
$$(x-3)^2+(y-4)^2=9+16,$$
$$(x-3)^2+(y-4)^2=(\sqrt{25})^2, \quad \_\_\_\_\_ \text{ eq.(1)}$$
$$\because\ (x-x_0)^2+(y-y_0)^2=r^2. \quad \_\_\_\_\_ \text{ eq.(2)}$$

The eq.(1) is now in the standard form of eq.(2). This equation represents a circle with the center at (3, 4) and with a radius equal to $\sqrt{25}$.

SOLUTION 8:

The standard form of equation of circle is

$(x-h)^2 + (y-k)^2 = r^2$,

Here $h=3$, $k=-2$, $r=4$,

$(x-3)^2 + (y-(-2))^2 = 4^2$,

$x^2-6x+9+y^2+4+4y=16$,

$x^2+y^2-6x+4y=16-9-4$,

$x^2+y^2-6x+4y=3.$

SOLUTION 9:

The distance formula between two points (x_1, y_1) and (x_2, y_2) in a coordinate plane is given by

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$,

$d=\sqrt{(8-2)^2+(6-4)^2}$,

$=\sqrt{(6)^2+(2)^2}$,

$=\sqrt{36+4}$,

$=\sqrt{40}$,

$=2\sqrt{10}.$

SOLUTION 10:

The radius is the distance between the center and any point on the circle, so find the distance:

$r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$,

$r=\sqrt{(3-(-1))^2+(-2-(-3))^2}$,

$=\sqrt{(3+1)^2+(-2+3)^2}$,

$=\sqrt{(4)^2+(1)^2}$,

$=\sqrt{16+1}$,

$=\sqrt{17}$,

$\approx 4.123.$

Then the radius is $\sqrt{17}$, or about 4.123, rounded to three decimal places.

Practice Exercise
Lecture No.6: Functions

Q.No.1

Find the natural domain and the range of the given function

$h(x) = \cos^2(\sqrt{x})$.

Solution:

As we know that the $\sqrt{x}$ is defined on non-negative real numbers $x \geq 0$. This means that the natural domain of $h(x)$ is the set of positive real numbers.

Therefore, the natural domain of $h(x) = [0, +\infty)$.

As we also know that the range of trigonometric function $\cos x$ is $[-1, 1]$.

The function $\cos^2 \sqrt{x}$ always gives positive real values within the range 0 and 1 both inclusive. From this we conclude that the range of $h(x) = [0, 1]$.

Q.No.2

Find the domain and range of function f defined by $f(x) = x^2 - 2$.

Solution:

$\because \quad f(x) = x^2 - 2$

The domain of this function is the set of all real numbers.

The range is the set of values that $f(x)$ takes as x varies. If x is a real number, x^2 is either positive or zero. Hence we can write the following:

$x^2 \geq 0$,

Subtract -2 on both sides to obtain

$x^2 - 2 \geq -2$.

The last inequality indicates that $x^2 - 2$ takes all values greater than or equal to -2. The range of function f is the set of all values of $f(x)$ in the interval $[-2, +\infty)$.

Q.No.3

Determine whether $y = \pm\sqrt{x+3}$ is a function or not? Justify your answer.

Solution:

$\because \quad y = \pm\sqrt{x+3}$

This is not a function because each value that is assigned to 'x' gives two values of y

So this is not a function. For example, if x=1 then

$y = \pm\sqrt{1+3}$,

$y = \pm\sqrt{4}$,

$y = \pm 2$.

Q.No.4

Determine whether $y = \dfrac{x+2}{x+3}$ is a function or not? Justify your answer.

Solution:

$\because \quad y=\frac{x+2}{x+3}$

This is a function because each value that is assigned to 'x' gives only one value of y
So this is a function. For example if x=1 then

$$y=\frac{1+2}{1+3},$$
$$y=\frac{3}{4},$$
$$y=0.75.$$

Q.No.5

(a) Find the natural domain of the function $f(x)=\frac{x^2-16}{x-4}$.

(b) Find the domain of function f defined by $f(x)=\frac{-1}{(x+5)}$.

Solution:

(a)

$$\because \quad f(x)=\frac{x^2-16}{x-4},$$
$$\Rightarrow \quad f(x)=\frac{(x+4)(x-4)}{(x-4)},$$
$$=(x+4) \quad ; \; x\neq 4.$$

This function consists of all real numbers x, except $x=4$.

(b)

$$\because \quad f(x)=\frac{-1}{(x+5)}$$

This function consists of all real numbers x, except $x=-5$. Since $x=-5$ would make the denominator equal to zero and the division by zero is not allowed in mathematics. Hence the domain in interval notation is given by $(-\infty,-5)\cup(-5,+\infty)$.

Practice Exercise
Lecture No.8: Graphs of Functions
Lecture No.9: Limits

Choose the correct option for the following questions:

1) If a vertical line insects the graph of the equation at two points, then which of the following is true.

I. It represents a function.

II. It represents a parabola.

III. It represents a straight line.

IV. It does not represent a function.
2) Which of the following is the reflection of the graph of about y-axis?
I.
II.
III.
IV.
3) Given the graph of a function and a constant c, the graph of can be obtained by
I. Translating the graph of up by c units.
II. Translating the graph of down by c units.
III. Translating the graph of right by c units.
IV. Translating the graph of left by c units.
4) Given the graph of a function and a constant c, the graph of can be obtained by
I. Translating the graph of up by c units.
II. Translating the graph of down by c units.
III. Translating the graph of right by c units.
IV. Translating the graph of left by c units.
5) Which of the following is the reflection of the graph of about x-axis?
V.
VI.
VII.
VIII.
Q.No.6
If and then find the value of 'c' so that .
Answer:
Q.No.7
Find the limit by using the definition of absolute value
Answer:
Q.No.8
Find the limit by using the definition of absolute value
Answer: -1

Q.No.9
Evaluate: .

Answer: 0

Q.No.10
Evaluate: .

Answer:

Practice Exercise
Lecture No.14: Rate of Change

Q.No.1

Find the instantaneous rate of change of $f(x) = x^2 + 1$ at x_0.

Answer: $2x_0$

Solution:

Since $f(x) = x^2 + 1$ at x_0,

$$\lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h\to 0} \frac{((x_0+h)^2+1)-(x_0^2+1)}{h},$$

$$= \lim_{h\to 0} \frac{x_0^2+h^2+2x_0h+1-x_0^2-1}{h},$$

$$= \lim_{h\to 0} \frac{h^2+2x_0h}{h},$$

$$= \lim_{h\to 0} \frac{h(h+2x_0)}{h},$$

$$= \lim_{h\to 0}(h + 2x_0),$$

$$= 2x_0$$ by applying limit, (Answer).

Q.No.2

Find the instantaneous rate of change of $f(x) = \sqrt{x+2}$ at an arbitrary point of the domain of f.

Answer: $\frac{1}{2\sqrt{a+2}}$

Solution:

Let a be any arbitrary point of the domain of f. The instantaneous rate of change of $f(x)$ at $x = a$ is

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{\sqrt{x+2}-\sqrt{a+2}}{x-a},$$

$$= \lim_{x\to a} \frac{\sqrt{x+2}-\sqrt{a+2}}{x-a} \times \frac{\sqrt{x+2}+\sqrt{a+2}}{\sqrt{x+2}+\sqrt{a+2}}$$ by rationalizing,

$$= \lim_{x\to a} \frac{x+2-a-2}{(x-a)\sqrt{x+2}+\sqrt{a+2}},$$

$$= \lim_{x\to a} \frac{x-a}{(x-a)\sqrt{x+2}+\sqrt{a+2}},$$

$$= \lim_{x\to a} \frac{1}{\sqrt{x+2}+\sqrt{a+2}},$$

$$= \frac{1}{\sqrt{a+2}+\sqrt{a+2}} \text{ by applying limit,}$$

$$= \frac{1}{2\sqrt{a+2}} \text{ (Answer).}$$

Q.No.3

The distance traveled by an object at time t is $= f(t) = t^2$. Find the instantaneous velocity of the object at $t_0 = 4\,sec$.

Answer: 8

Solution:

$$v_{inst} = m_{tan} = \lim_{t_1\to t_0} \frac{f(t_1)-f(t_0)}{t_1-t_0},$$

$$= \lim_{t_1\to t_0} \frac{t_1^2-4^2}{t_1-t_0},$$

$$= \lim_{t_1\to t_0} \frac{t_1^2-16}{t_1-t_0},$$

$$= \lim_{t_1\to t_0} \frac{(t_1+4)(t_1-4)}{t_1-t_0},$$

$$= \lim_{t_1\to 4} \frac{(t_1+4)(t_1-4)}{(t_1-4)} \text{ because } t_0 = 4\,sec,$$

$$= \lim_{t_1\to 4}(t_1+4),$$

$$= 4+4 \text{ by applying limit,}$$

$$= 8 \text{ (Answer).}$$

Q.No.4

Find the instantaneous rate of change of $f(x) = x^3 + 1$ at $x_0 = 2$.

Answer: 12

Solution:

$$\lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h},$$

$$= \lim_{h\to 0} \frac{((2+h)^3+1)-(2^3+1)}{h},$$

$$= \lim_{h\to 0} \frac{(2^3+3(2)^2h+3(2)h^2+h^3+1)-(2^3+1)}{h},$$

$$= \lim_{h\to 0} \frac{8+12h+6h^2+h^3+1-(8+1)}{h},$$

$$= \lim_{h\to 0} \frac{9+12h+6h^2+h^3-9}{h},$$

$$= \lim_{h\to 0} \frac{12h+6h^2+h^3}{h},$$

$$= \lim_{h\to 0} \frac{h(12+6h+h^2)}{h},$$

$$= \lim_{h\to 0} (12 + 6h + h^2),$$

$$= 12 \text{ (Answer)}.$$

Q.No.5

(a) The distance traveled by an object at time t is $s = f(t) = t^2$. Find the average velocity of the object between $t = 2\ sec.$ and $t = 4\ sec.$

(b) Let $f(x) = \frac{1}{x-1}$. Find the average rate of change of f over the interval [5,7].

Answer: (a) 14 **(b)** $\frac{5}{2}\ m/sec$

Solution:

(a) Avergae Velocity $= \frac{Distance\ travelled\ during\ interval}{TIme\ Elapsed},$

$$v_{ave} = \frac{f(t_1)-f(t_0)}{t_1-t_0},$$

$$= \frac{f(4)-f(2)}{4-2},$$

$$= \frac{4^2-2^2}{2},$$

$$= \frac{16-4}{2},$$

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$$= \frac{12}{2},$$

$$= 6 \text{ (Answer).}$$

(b) Avergae Velocity $= \frac{Distance\ travelled\ during\ interval}{TIme\ Elapsed},$

$$m_{sec} = \frac{f(x_1)-f(x_0)}{x_1-x_0},$$

$$= \frac{f(7)-f(5)}{7-5},$$

$$= \frac{\frac{1}{7-1}-\frac{1}{5-1}}{2},$$

$$= \frac{\frac{1}{6}-\frac{1}{4}}{2},$$

$$= -\frac{1}{24}\ m/sec. \text{ (Answer).}$$

Practice Exercise

Lecture No.17: Derivatives of Trigonometric Functions

Q.No.1

Find $\frac{dy}{dx}$ if $y = x^3 \cot x - \frac{3}{x^3}$.

Answer: $3x^2 \cot x - x^3 cosec^2 x + \frac{9}{x^4}$

Solution:

Given $y = x^3 \cot x - \frac{3}{x^3},$

$$\frac{dy}{dx} = \cot x \frac{d}{dx}(x^3) + x^3 \frac{d}{dx}(\cot x) - \frac{d}{dx}\left(\frac{3}{x^3}\right),$$

$$= \cot x\,(3x^2) + x^3(-cosec^2 x) - 3\frac{d}{dx}\left(\frac{1}{x^3}\right),$$

$$= 3x^2 \cot x - x^3 cosec^2 x + \frac{9}{x^4} \quad \text{(Answer).}$$

Q.No.2

Find $\frac{dy}{dx}$ if $y = x^4 \sin x$ at $x = \pi$.

Answer: $-\pi^4$

Solution:

$\because \quad \frac{d}{dx}(f.g) = f.\frac{d}{dx}(g) + g.\frac{d}{dx}(f),$

$y = x^4 \sin x$ at $x = \pi,$

$\frac{d}{dx} = \sin x \frac{d}{dx}(x^4) + x^4 \frac{d}{dx}(\sin x),$

$= \sin x \ (4x^3) + x^4(\cos x),$

$= 4x^3 \sin x + x^4 \cos x,$

$= 4\pi^3 \sin \pi + \pi^4 \cos \pi,$ at $x = \pi,$

$= 4\pi^3(0) + \pi^4(-1),$

$= -\pi^4$ (Answer).

Q.No.3

Find $f'(t)$ if $f(t) = \frac{2-8t+t^2}{sint}$.

Answer: $\frac{[(2t-8)(\sin t)]-[(t^2-8t+2)(\cos t)]}{\sin^2 t}$

Solution:

$\because \ \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x).\frac{d}{dx}(f(x)) - f(x).\frac{d}{dx}(g(x))}{[g(x)]^2},$

$f(t) = \frac{2-8t+t^2}{sint},$

$f'(t) = \frac{[(\sin t)(-8+2t)]-[(2-8t+t^2)(\cos t)]}{(\sin t)^2},$

$= \frac{[(2t-8)(\sin t)]-[(t^2-8t+2)(\cos t)]}{\sin^2 t}$ (Answer).

Q.No.4

Find $f'(y)$ if $(y) = \frac{\sin y + 3\tan y}{y^3-2}$.

Answer: $\frac{[(y^3-2)(\cos y + 3\sec^2 y)]-[(\sin y+3\tan y)+(3y^2)]}{y^6-4y^3+4}$

Solution:

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x).\frac{d}{dx}(f(x)) - f(x).\frac{d}{dx}(g(x))}{[g(x)]^2},$

$f(y) = \frac{\sin y + 3\tan y}{y^3-2}.$

$f'(y) = \frac{[(y^3-2)(\cos y + 3\sec^2 y)]-[(\sin y+3\tan y)+(3y^2)]}{(y^3-2)^2},$

$= \frac{[(y^3-2)(\cos y + 3\sec^2 y)]-[(\sin y+3\tan y)+(3y^2)]}{y^6-4y^3+4}$ (Answer).

Q.No.5

(a) Find $\frac{dy}{dx}$ if $y = (5x^2 + 3x + 3)(\sin x)$.

(b) Find $f'(t)$ if $(t) = 5t \sin t$.

Answer: **(a)** $(5\,x^2+3x+3)(\cos x)+\sin x\,.(10x+3)$
(b) $5\,t\cos t+5\sin t$

Solution:

(a) $\because\ \frac{d}{dx}(f.g)=f.\frac{d}{dx}(g)+g.\frac{d}{dx}(f),$

$y=(5x^2+3x+3)(\sin x)\,,$

$\frac{d}{dx}[(5\,x^2+3x+3)(\sin x)]=(5\,x^2+3x+3)(\cos x)+\sin x\,(10x+3)$ (Answer).

(b) $\because\ \frac{d}{dx}(f.g)=f.\frac{d}{dx}(g)+g.\frac{d}{dx}(f),$

$f(t)=5t\sin t\,,$

$\frac{d}{dx}(5\,t\sin t)=5\,t\cos t+(\sin t)(5),$

$=5\,t\cos t+5\sin t$ (Answer).

Practice Exercise

Lecture No.20: Derivatives of Logarithmic and Exponential Functions

Q.No.1

Differentiate: $y=(5-x)^{\sqrt{x}}$.

Answer: $\frac{dy}{dx}=\left(\frac{\ln(5-x)}{2\sqrt{x}}-\frac{\sqrt{x}}{(5-x)}\right).(5-x)^{\sqrt{x}}$

Solution:

$\because \quad y=(5-x)^{\sqrt{x}}$,

taking log on both sides ,

$\Rightarrow \ln y=\sqrt{x}\,\ln(5-x) \qquad (\because \ln m^n = n\ln m)$,

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{1}{2\sqrt{x}}.\ln(5-x)+\frac{1}{(5-x)}(-1).\sqrt{x}$,

$\Rightarrow \frac{dy}{dx}=\left(\frac{\ln(5-x)}{2\sqrt{x}}-\frac{\sqrt{x}}{(5-x)}\right).y$,

$\Rightarrow \frac{dy}{dx}=\left(\frac{\ln(5-x)}{2\sqrt{x}}-\frac{\sqrt{x}}{(5-x)}\right).(5-x)^{\sqrt{x}}$.

Q.No.2

Differentiate $y=(\cos x)^{8x}$ with respect to 'x'.

Answer: $\frac{dy}{dx}=\left(8\ \ln(\cos x)-\frac{8x\sin x}{\cos x}\right)(\cos x)^{8x}$

Solution:

$\because \quad y=(\cos x)^{8x}$,

taking log on both sides,

$\Rightarrow \ln y=(8x)\ln(\cos x)$, $\qquad (\because \ln m^n = n\ln m)$,

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=8.\ln(\cos x)+\frac{1}{(\cos x)}.(-\sin x).(8x)$, $\qquad \left(\because \frac{d}{dx}(\ln x)=\frac{1}{x}\ ;\ \frac{d}{dx}(\cos x)=-\sin x\right)$,

$\Rightarrow \frac{dy}{dx}=\left(8\ \ln(\cos x)-\frac{8x\sin x}{\cos x}\right).y$,

$\Rightarrow \frac{dy}{dx}=\left(8\ \ln(\cos x)-\frac{8x\sin x}{\cos x}\right)(\cos x)^{8x}$.

Q.No.3

Differentiate $y=x^{\sin 5x}$ with respect to 'x'.

Answer: $\frac{dy}{dx}=\left(5(\cos 5x).\ln(x)+\frac{\sin 5x}{x}\right)\left(x^{\sin 5x}\right)$

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Solution:

$\because \quad y = x^{\sin 5x}$,

Taking log on both sides ,

$\Rightarrow \ln y = (\sin 5x)\ln(x)$, $\quad (\because \ln m^n = n\ln m)$,

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 5(\cos 5x).\ln(x) + \frac{1}{x}.(\sin 5x)$, $\quad \left(\because \frac{d}{dx}(\ln x) = \frac{1}{x}\ ;\ \frac{d}{dx}(\sin x) = \cos x\right)$,

$\Rightarrow \frac{dy}{dx} = \left(5(\cos 5x).\ln(x) + \frac{\sin 5x}{x}\right).y$,

$\Rightarrow \frac{dy}{dx} = \left(5(\cos 5x).\ln(x) + \frac{\sin 5x}{x}\right)\left(x^{\sin 5x}\right)$.

Q.No.4

Differentiate $y = x\,e^{3x+4}$.

Answer: $\frac{dy}{dx} = e^{3x+4} + 3xe^{3x+4}$

Solution:

$\because \quad y = x\,e^{3x+4}$,

$\Rightarrow \frac{dy}{dx} = e^{3x+4} + xe^{3x+4}\frac{d}{dx}(3x+4)$,

$\Rightarrow \frac{dy}{dx} = e^{3x+4} + 3xe^{3x+4}$.

Q.No.5

Find the derivative of the function $y = \ln(2 + x^5)$ with respect to 'x'.

Answer: $\frac{dy}{dx} = \frac{5x^4}{(2 + x^5)}$

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Solution:

$y = \ln(2 + x^5)$,

now taking the derivative of the function on both sides ,

$$\frac{dy}{dx} = \frac{d}{dx}\{\ln(2 + x^5)\} ,$$

$$\frac{dy}{dx} = \frac{1}{(2 + x^5)} \frac{d}{dx}(2 + x^5) ,$$

$$\frac{dy}{dx} = \frac{1}{(2 + x^5)}(0 + 5x^4) ,$$

$$\frac{dy}{dx} = \frac{5x^4}{(2 + x^5)} .$$