MTH101 Calculus And Analytical Geometry Lecture Wise Questions and Answers For Final Term Exam Preparation

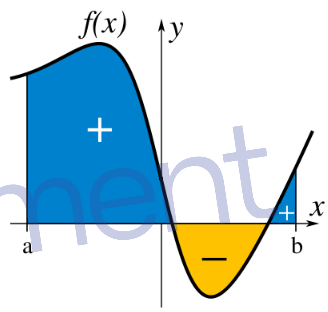
Lecture No 23 to 45

Complete and Important Question and answer

1. What is the difference between definite integral and an indefinite integral?

If the function f is continuous on $[a, b]$, and can assume both positive and negative values, the **definite integral**

$$\int_a^b f(x)\,dx$$



is net signed area between $y = f(x)$ and the interval $[a, b]$. The numbers a and b are called the lower and upper limits of integration.

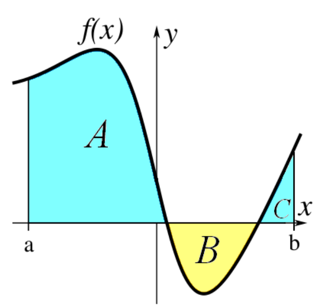
Indefinite integral is the set of functions *F(x) + C*, where *C* is constant of integration *and F(x)* is the integral of given function f where $\dfrac{d\left[F(x)\right]}{dx} = f(x)$.

Q 2. What is meant by net signed area (the term used in defining definite integral)?

An indefinite integral is of the form

$$\int f(x)\,dx = F(x) + C$$

Suppose that a function f has a smooth curve in an interval [a, b] and can lie both above and below x-axis then 'area under the curve and above x-axis' minus 'area above the curve below x-axis' is termed as net signed area under the curve of f on the interval [a, b]. It is named "signed" area because the area above the x -axis counts as positive and the area below the x -axis counts as negative.Net signed area can be positive, negative or zero; it is positive when there is more area above than below, negative when there is more area below than above, and zero when the area above and below are equal.For example, in the figure



Net signed area under the curve of f on the interval [a, b]

= [Area above] – [Area below]

= (A + C) – B

Q 3. Why we add C with the anti-derivative for evaluating an indefinite integral?

ANSWER: If a function $f(x)$ is defined on an interval and $F(x)$ is an antiderivative of $f(x)$, then the indefinite integral is set of *all* anti-derivatives of $f(x)$, that is, the functions $F(x) + C$, where C is an arbitrary constant.

As we know, the derivative of any constant function is zero. Once one has found one antiderivative *F(x)*, adding or subtracting a constant C will give us another antiderivative, because $(F(x) + C)' = F'(x) + C' = F'(x)$. The constant is a way of expressing that every function has an infinite number of different anti-derivatives.

For example, suppose one wants to find anti-derivatives of $\cos(x)$. One such anti-derivative is $\sin(x)$. Another one is $\sin(x) + 1$. A third is $\sin(x) - \pi$. Each of these has derivative $\cos(x)$, so they are all anti-derivatives of $\cos(x)$.

It turns out that adding and subtracting constants is the only flexibility we have in finding different anti-derivatives of the same function. That is, all anti-derivatives are the same up to a constant. To express this fact for cos(x), we write:

$$\int \cos(x)\,dx = \sin(x) + C$$

Q 6. State First Fundamental Theorem of Calculus?

The First Fundamental Theorem of Calculus states that:

If f is continuous on the closed interval $[a, b]$ and F is the anti-derivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Q.1# Why we use the second fundamental theorem of calculus?

Answer: The first fundamental theorem of calculus is used to evaluate the definite integral of a continuous function if we can find an antiderivative for that function, but it does not point out the question of which functions actually have antiderivatives; then at this stage the second fundamental theorem of calculus is used.

The **second fundamental theorem of calculus** states that the derivative of a definite integral with respect to its upper endpoint is its integrand; it allows one to compute the definite integral of a function by using any one of its infinitely many antiderivatives. This part of the theorem has invaluable practical applications, because it markedly simplifies the computation of definite integral.

Q.2# Why the function defined by $F(x)=\int_a^x f(t)\,dt$ is not an arbitrary antiderivatives of f ; if $x=a$?

Answer: The function defined by $F(x)=\int_a^x f(t)\,dt$ is not an arbitrary antiderivative of f ; it is the specific antiderivative whose value at $x=a$ is zero because $F(a)=\int_a^a f(t)\,dt=0$

Q 1. What is the difference between local maximum and relative maximum value of a function?

ANSWER: Local maximum and relative maximum are synonyms, that is, different words for same concept. Similarly local minimum is also named as relative minimum.

Q 2. What is the difference between critical point and stationary point of the function?

ANSWER: The term "critical point" is often confused with "stationary point".

Stationary point x_0 is the point, at which the derivative of a function $f(x)$ vanishes,
$f'(x_0) = 0$
That is, a point where the function "stops" increasing or decreasing (hence the name).

Critical point x_0 of a function $y = f(x)$ is the point at which $f'(x_0) = 0$

Or

$f(x)$ is not differentiable at x_0

Critical point is more general, it is *either* a stationary point *or* a point where the derivative is not defined.

A stationary point is always a critical point, but a critical point is not always a stationary point as it may also be a non-differentiable point.

Q.1# What is the difference between a sequence and series?

Answer:

A **sequence** is a number pattern in a definite order following a certain rule.

Examples of sequences:

1) 1, 2, 3, 4, 5, 6, 7, ... *add 1 to the preceding term*

2) 2, 4, 7, 11, 16, 23, 31. *add 2 to the preceding term, add 3 to the next term, etc*

3) 1, 1, 2, 3, 5, 8, 13, 21, 34,... *add the two preceding terms together*

A **sequence** is an ordered list of objects (or events). Like a set, it contains members (also called *elements* or *terms*), and the number of terms (possibly infinite) is called the *length* of the sequence. Unlike a set, order matters, and the exact same elements can appear multiple times at different positions in the sequence.

For example, (C, R, Y) is a sequence of letters that differs from (Y, C, R), as the ordering matters. Sequences can be *finite*, as in this example, or *infinite*, such as the sequence of all even positive integers (2, 4, 6,...).

General sequence terms are usually denoted by:

$\mathbf{a}_1, \mathbf{a}_{2,\ldots}\mathbf{a}_n, \mathbf{a}_{n+1\ldots}$

$\mathbf{a}_1$ 1st term

$\mathbf{a}_2$ 2nd term

$\mathbf{a}_n$ nth term

$\mathbf{a}_{n+1}$ (n+1) term

A **series** is a sum of terms in a sequence. It can be denoted by

$$\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4 + \ldots$$

Using the above sequences, we have the following series:

1) 1 + 2 + 3 + 4 + 5 + 6 + 7 +...

2) 2 + 4 + 7 + 11 + 16 + 23 + 31.

3) 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 + 34 +...

Q.2# At what condition a sequence is called monotone?

Answer: A sequence which is either always increasing or always decreasing is called a *monotone* sequence. Note that an ``arbitrary'' sequence is not monotone (it will usually sometimes increase, and sometimes decrease).

Q.3# How do we use monotone sequence in real life?

Answer: Monotone sequences do happen in real life. For example, the sequence

$a_1 = 3$ $a_2 = 3.1$ $a_3 = 3.14$ $a_4 = 3.141$ $a_5 = 3.1415...$ is how we often describe the decimal expansion of π. Monotone sequences are important because we can say something useful about them which are not true of more general sequences.

Q.4# In what disciplines infinite series can be used?

Answer: Infinite series are widely used in other quantitative disciplines such as physics and computer science.

Q.5# When a sequence converges or diverges?

Answer: A sequence **converges** to a limit L if and only if the sequence of even numbered terms and odd numbered terms both converges to L e.g. The sequence 1/2, 1/3, $1/2^2$, $1/3^2$, $1/2^3$, $1/3^3$, ... converges to 0. Note that the even numbered terms and odd numbered terms both converges to 0.(mean as the terms of the sequence increases they become closer to zero, i.e. 1/2 =0.5 , $1/2^2$= 0.25, $1/2^3$= 0.125, 0.0625, 0.03125, 0.015625, 0.0078125,....as the number of term increases it becomes closer to zero).

If both or one of even numbered terms and odd numbered terms are not converges to limit L then the sequence **diverges**.

Q.6# How do we check the convergent or divergent for series?

Answer: For series we have many methods to check that either the series is convergent or divergent. We have Divergence test, Integral test, convergence of p-Series, Comparison test, Ratio test, the Root test, the Limit comparison Test.

There are a number of methods of determining whether a series converges or diverges

The terms of the sequence $\{a_n\}$ are compared to the sequence of $\{b_n\}$ for all n

Where $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ converge then so $\sum_{n=1}^{\infty} a_n$

Similarly if

For all $0 \le b_n \le a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges then so $\sum_{n=1}^{\infty} a_n$

Ratio Test:

Assume that for all n $a_n > 0$

Suppose that there exist "r" such that $\lim_{x\to\infty} |\frac{a_{n+1}}{a_n}| = r$

If r < 0 then the series converges if r >1 then the series diverge

If r =1 then the series may converge or diverge

Similarly we can apply the other tests such as the Integral test, the Limit comparison test; the Alternating series test to check whether the given series converges or diverges.

Q.7# How do we write an alternating series?

Answer: An **alternating series** is any series, $\sum a_n$, for which the series terms can be written in one of the following two forms.

$$a_n = (-1)^n b_n \qquad b_n \geq 0$$
$$a_n = (-1)^{n+1} b_n \qquad b_n \geq 0$$

There are many other ways to deal with the alternating sign, but they can all be written as one of the two forms above. For instance,

$$(-1)^{n+2} = (-1)^n (-1)^2 = (-1)^n$$
$$(-1)^{n-1} = (-1)^{n+1} (-1)^{-2} = (-1)^{n+1}$$

Q.8# When a series is called a converge conditionally?

Answer: A series $\sum_{n=0}^{\infty} a_n$ is said to **converge conditionally** if $\lim_{m\to\infty} \sum_{n=0}^{m} a_n$ exists and is a finite number (not ∞ or −∞), but $\sum_{n=0}^{\infty} |a_n| = \infty.$

A classical example is given by

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$$

which converges to ln 2 , but is not absolutely convergent.

The simplest examples of conditionally convergent series (including the one above) are the alternating series.

Answer:

Q.1# Where definite integral is applicable?

Answer: There are many applications of definite integral for example: it is used to find the area bounded by curves, area of a surface of revolution, volume of solids (i.e. volume by slicing and washers as well as cylindrical shell), length of plane curves, work, mass and fluid pressure. We use definite integrals to solve many practical problems.

Q.2# How can we find the volume of a disc when it has some definite thickness?

Answer: When we find the area of disc we suppose that the thickness of the disc is zero and at that time area of the disc is πr^2. This area is also called the surface area of the disc and in this case since disc is in 2 dimensions, so its volume is zero. Now if there is some thickness of the disc say h. Then its volume is simply multiplication of its surface area and thickness that is $\pi h r^2$.

Q.3# Either cylinders can consider as hollow or solid?

Answer: A cylinder refers to a solid bounded by a cylindrical surface and two parallel planes. Hollow cylinder is named as cylindrical surface.

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Q.4# How do we calculate the volume using shells?

Answer:: A shell is a hollow tube. To calculate volume using shells, we take shells of graduated radii and fill the solid of revolution with them. We only want the volume of the material in each shell, not what the shell itself might hold. We calculate the volume of each shell and add them all up to get the total volume.

The formula for the volume of a shell is $V = 2 \pi r h w$
w is our *dx* or *dy*. The width and the height of the shell tell us which to use.

Q.5# How do we find the length of a curve?

Answer: Simply we find the length of a given curve by dividing the curve into very small segments and calculating the length of each segment. Then we add up the lengths found.

Mathematically we can say that measure the intervals by keeping the Δx same for each interval. Each segment approximates a straight line, so use the distance formula to find the length of each segment.

Question: FAQs on Improper Integral (Lecture #39)

Answer:

Lecture # 39   Improper Integral

Q.1# What type of an integral is called when the integrand is not bounded and do not exist over a finite interval?

Answer: An integral exists if the integrand is both "bounded" and exists over a "finite" interval. If either of these conditions is not true then the integral is said to be "improper".

Q.2# Why do we replace the infinity with a variable solving the improper integrals?

Answer: Such type of integrals in which one or both of the limits of integration are infinity, the interval of integration is said to be over an infinite interval. We will replace the infinity with a variable (usually ***t***), do the integral and then take the limit of the result as ***t*** goes to infinity.

Q.3# When the limit of improper integral converges or diverges?

Answer: The value of the limits is the value assigned to the integral. If this limit exists, the improper integral is said to be **converge** i.e. we will call these integrals convergent if the associated limit exists and is a finite number (*i.e.* it is not plus or minus infinity).

If the limit does not exist, then the improper integral is called to **diverge**, in which case it is not assigned a value i.e. divergent if the associated limits either doesn't exist or is (plus or minus) infinity.

Lecture # 40 L'Hopital's Rule and Indeterminate Forms

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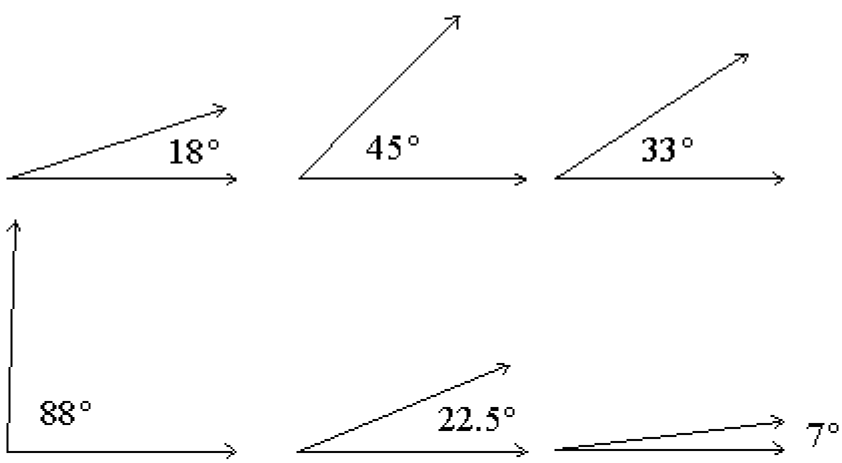
Q :Define cute angel

Acute Angles

An acute angle is an angle measuring between 0 and 90 degrees.

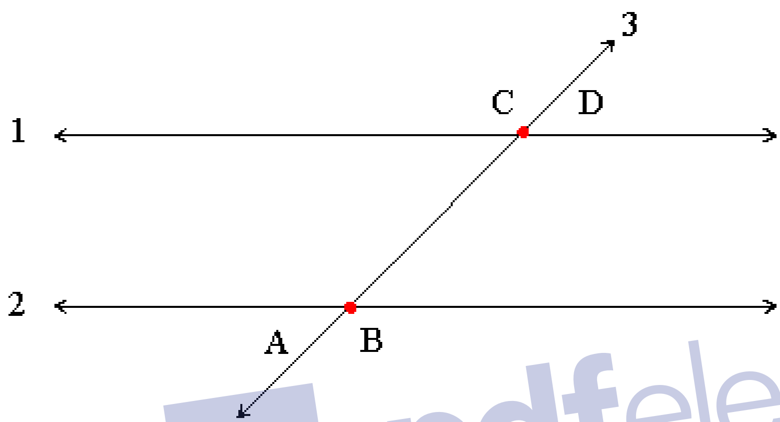
Example:

The following angles are all acute angles.



Q: What is Alternate Exterior Angles

For any pair of parallel lines 1 and 2, that are both intersected by a third line, such as line 3 in the diagram below, angle A and angle D are called alternate exterior angles. Alternate exterior angles have the same degree measurement. Angle B and angle C are also alternate exterior angles.



Define Alternate Interior Angles

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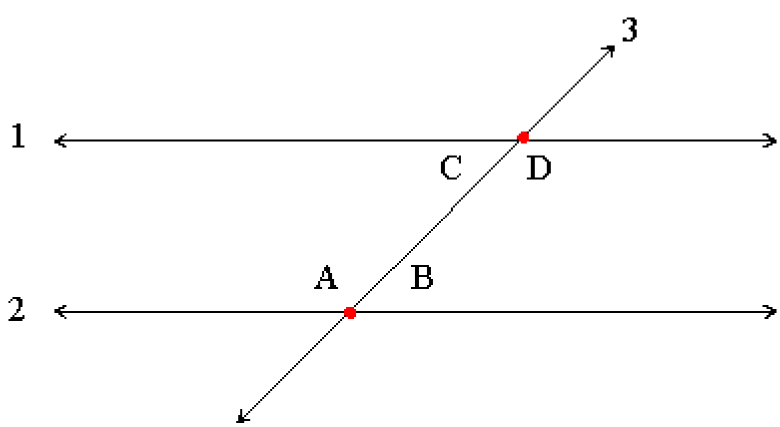
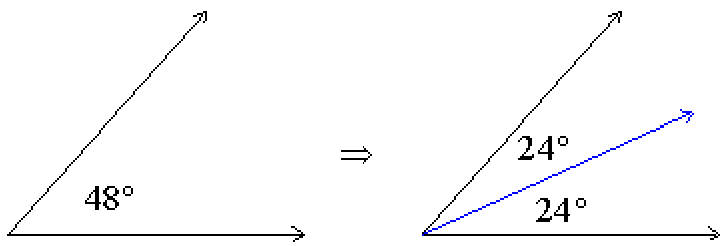


Define Angle Bisector

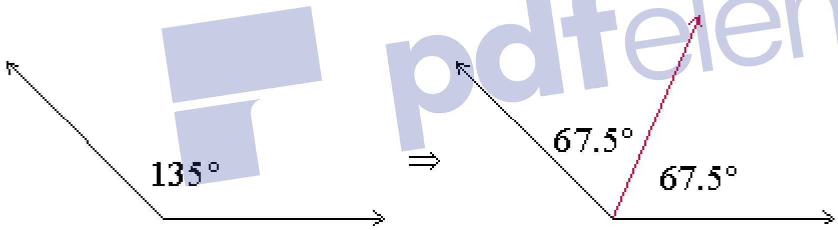
An angle bisector is a ray that divides an angle into two equal angles.

Example:

The blue ray on the right is the angle bisector of the angle on the left.



The red ray on the right is the angle bisector of the angle on the left.

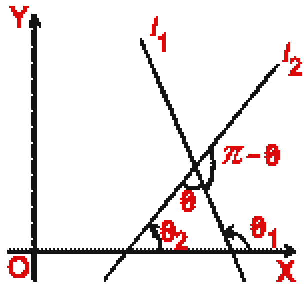


Q: Define Angle of inclination

The angle of inclination:

The positive angle, less than 180°, measured from the positive .x-axis to the given line.

Let l_1 and l_2 be the two non-vertical and non-perpendicular lines with slopes m_1 and m_2 respectively. Let θ_1 and θ_2 be their inclinations, then $m_1 = \tan \theta_1$ and $m_2 = \tan \theta_2$. There are two angles $\theta$and $\pi - \theta$ between the lines l_1 and l_2, given by tan $\theta = \pm (m_1 - m_2)(1 + m_1 m_2)$



This angle $\theta = \tan^{-1}(\frac{(m_1 - m_2)}{1 + m_1 m_2})$ is always between 0° and 180°, and is measured

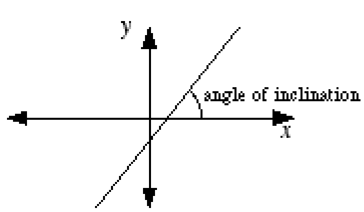
counterclockwise from the part of the *x*-axis to the right of the line.

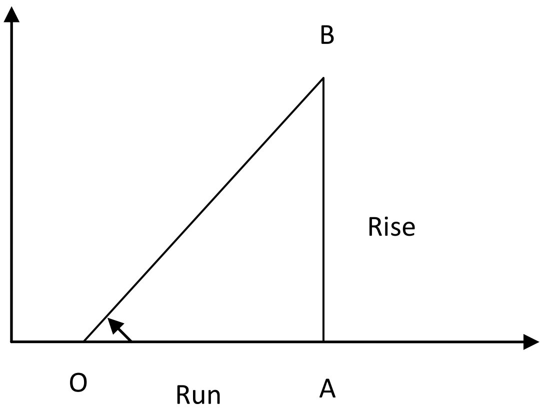
Q: define Angle of Inclination of a Line

Angle of Inclination of a Line

The angle between a line and the *x*-axis. This angle is always between 0° and 180°, and is measured counter clock wise from the part of the *x*-axis to the right of the line.

Note: All horizontal lines have angle of inclination 0°. All vertical lines have angle of inclination 90°. Also, the slope of a line is given by the tangent of the angle of inclination.





In the above figure, if we take slope of the line , then it will become

$$m = \frac{AB}{OA} \text{ ....(i)}$$

In the above figure, you can see that if we take tan θ , then it will become

$$\text{Tan } \Phi = \frac{AB}{OA} \text{ .... (ii)}$$

From (i) and (ii), we can equate the equations, so we have

$$m = \tan \Phi$$

FAQ on Circle problem

Let P(2,3) be a point on circle with center (4,-1). Find the slope of the line that is tangent to the circle at P.Here we need a diagram. But let me tell you about tangent to the circle; A tangent is a line that touches the circle only on a single point. Consider the

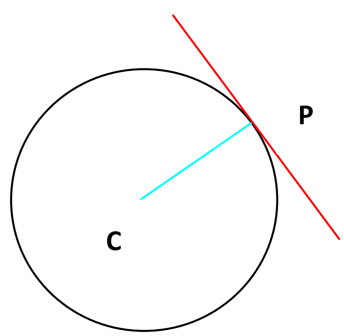


diagram:

Let C be the center of the circle and P be the given point on the circle. Then we can find the slope of the line CP and then also that of the perpendicular line that is the tangent to the circle at P (shown with red color, tangent is always perpendicular to such line).

You solved as:

Let P(2,3) the point on circle with center C(4,-1) then the slope of line CP is given as

$$m = \frac{-1-3}{4-2} = -2$$

This is the slope of the line CP , now we are to find the slope of the tangent line. As this is perpendicular to the line CP so its slope is

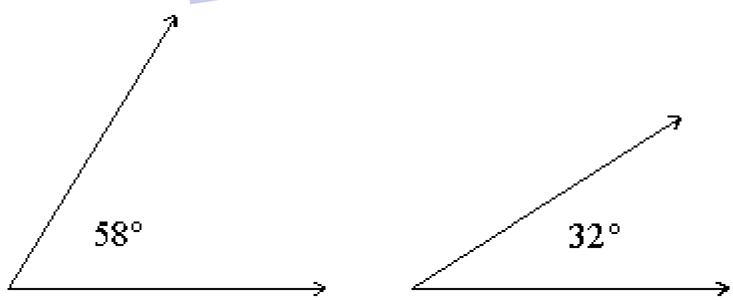
-1/m= ½

Q: Define Complementary Angles

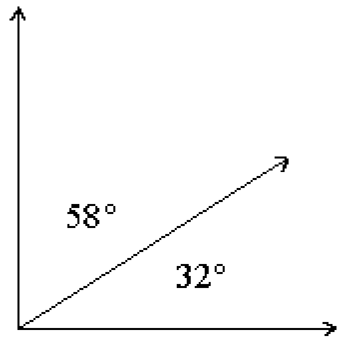
Two angles are called complementary angles if the sum of their degree measurements equals 90 degrees. One of the complementary angles is said to be the complement of the other.

Example:

These two angles are complementary.



Note that these two angles can be "pasted" together to form a right angle!



What is Complex Numbers?

The number of the form z = a + ib where a and b are real numbers and “i” read as iota.

“a” is called the real part of the complex number and “b” is called the imaginary part of the complex number.

Every real number can be written in the form of complex number with 0 imaginary part.

For example 5 is a real number it become a complex number in this way 5 + 0i. Here 5 is the real part and 0 is the imaginary part.

Real numbers is a subset of complex numbers.

The set of complex number is denoted by C.

The complex number z = a + ib can be written as (a, b).

Define Composition of function ?

In simple words:

When we operate an operator (operator may be addition or subtraction or any other) on any two functions we get a new function, that's mean we compose a new function.

For example:-

If we have two functions

F(x)=1+$\sqrt{x-2}$ and g(x)=x-1

If we use addition operator between two functions then, we have

F(x)+ g(x) =(1+$\sqrt{x-2}$)+(x-1)

=1+$\sqrt{x-2}$ + x-1

=x+$\sqrt{x-2}$

(That's mean we have compose a new function by adding two functions)

Another Example:-

If we have two functions

F(x)= x^3 and g(x)=x+4

(F$\circ$g)(x)=f(g(x)) [By definition]

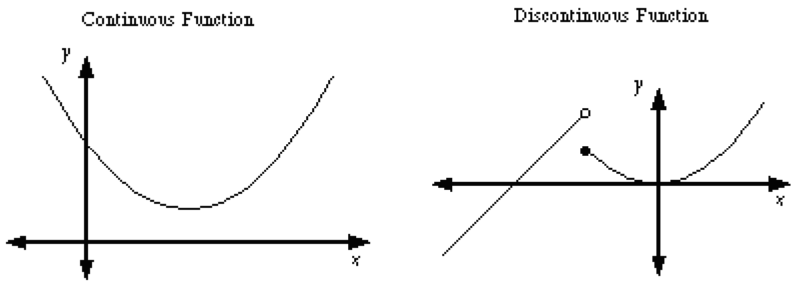
=f(x+4)[since g(x) =x+4]

=$(x+4)^3$ [We have replace x by x+4 in F(x) since F(x) = x^3]

That's mean we have compose a new function

Q: What are Continuous Function?

A function with a connected graph.

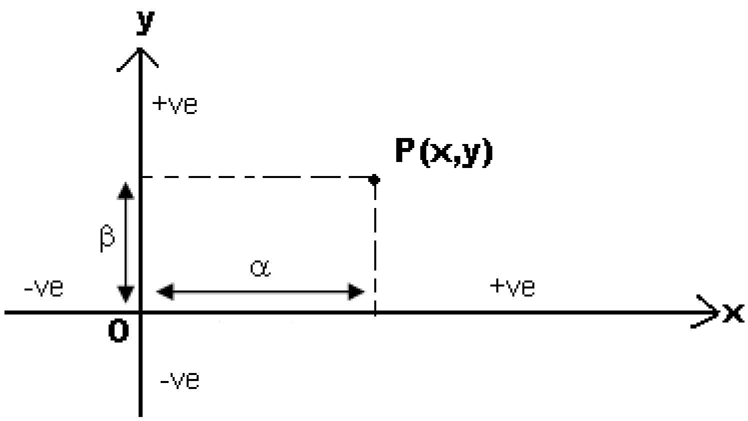


Definition of Continuity: A function is continuous at $x = a$ if

1. $\lim_{x \to a} f(x)$ exists
2. $f(a)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

Q: What's meant by Coordinates Lines?

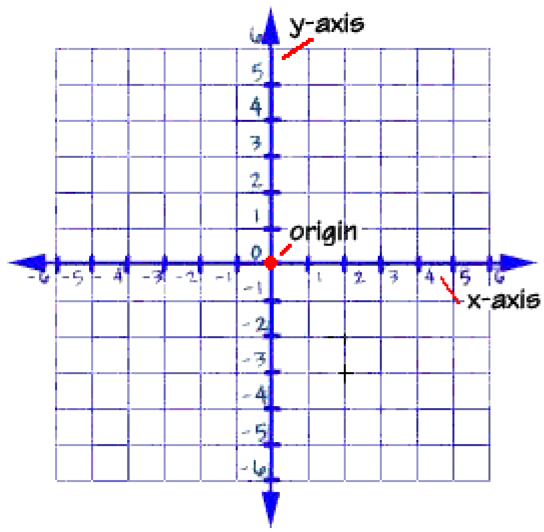
In coordinate geometry, ox and oy are **two lines** intersecting at right angles at O, where Ox and Oy are the **coordinate axes or Coordinates Lines** and O is the origin. Any point lying to the right of Oy has a positive x-coordinate and any point above the Ox axis has a positive y-coordinate as indicated in fig below. Any point P can be determined if its distance from the x and y axis are given. The distance of a point P from the y-axis is called the x-coordinate of the point and its distance from the x-axis, the y-coordinate.



A point P whose x-coordinate is α and y-coordinate β is represented by P(α,β).

Q: What are Coordinate Plane:

To draw a coordinate plane, start with a sheet of graph or any plane paper. Next, draw two lines perpendicular to each passing through origin, making angle of 90 degree with each other.



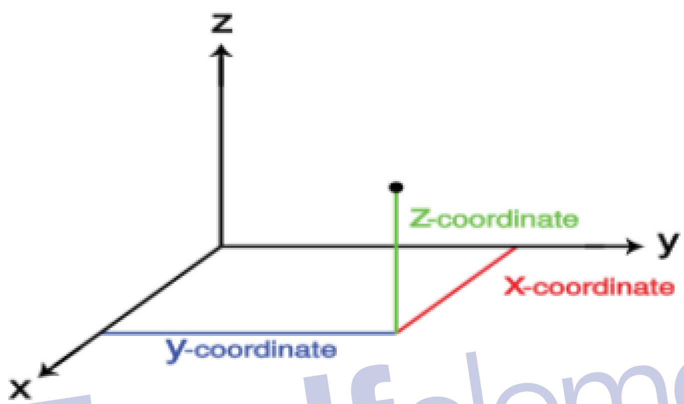
The line along the horizontal is called x-axis and the vertical line is called y-axis. The point where the x and y axes intersect is called the origin.

Locating Points:

We can locate any point on the coordinate plane as shown in figure. For example, to graph the point P(4, 2) we count right along the x axis 4 units, and then count up 2 units. Be careful to always start with the x axis, the point (4,2) is very different than the point (2,4)

Q: Define coordinate system

In analytical geometry, we can describe every point in three-dimensional space by means of three coordinates. Three coordinate axes make right angle with the other two at their mutual crossing point, called the origin .They are usually labeled as *x*, *y*, and *z*. Relative to these axes, the position of any point in three-dimensional space can be given by an ordered triple (x,y,z) of real numbers such as (3,5,7) or (-5,6)-8), each number giving the distance of that point from the origin measured along the axis.



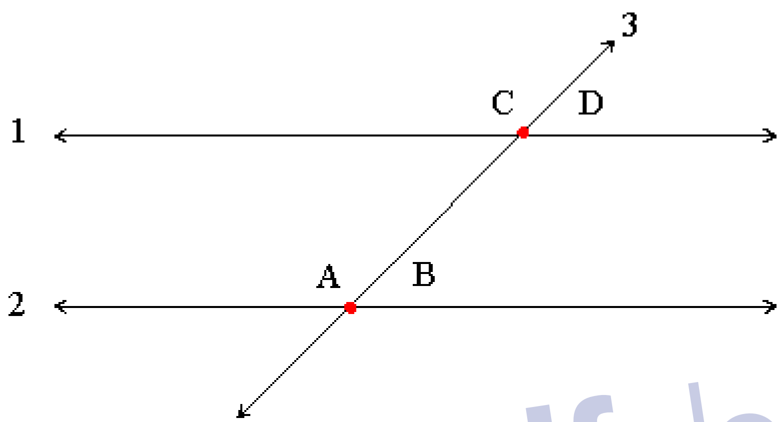
Coordinate systems

We can locate points in a given space by means of numerical quantities specified with respect to some frame of reference such as origin. These quantities are the coordinates of a point. To each set of coordinates, there corresponds just one point in any coordinate system.

A Cartesian coordinate system is one of the simplest and most useful systems of coordinates. It is constructed by choosing a point "*O*" designated as the origin. Through it, there are three intersecting directed lines *OX*, *OY*, *OZ*, the coordinate axes. The coordinates of a point *P* are (x, *y*, z). Usually the three axes are taken to be mutually perpendicular, in which case, the system is known as rectangular Cartesian.

Q:Define Corresponding Angles

For any pair of parallel lines 1 and 2, that are both intersected by a third line, such as line 3 in the diagram below, angle A and angle C are called corresponding angles. Corresponding angles have the same degree measurement. Angle B and angle D are also corresponding angles.



Simple Example of Derivate

The derivate of $(x)^2$

Differentiate it with respect to "x"

First we apply the Power Rule, and then we differentiate the function with respect to the given variable

$$= 2x\frac{dx}{dx}$$

= 2x (1)

= 2x

here 2x comes by power rule, and then we differentiate the function""(x)"" with respect to the given variable "x" which is equal to 1

Now

The derivate of $(y)^2$

Differentiate it with respect to “x”

$$= 2y\frac{dy}{dx}$$

here 2y comes by power rule, and then we differentiate the function””(y)”” with respect to the given variable “x” which become $\frac{dy}{dx}$

If

The derivate of $(y)^2$

Differentiate it with respect to “y”

Then its answer comes

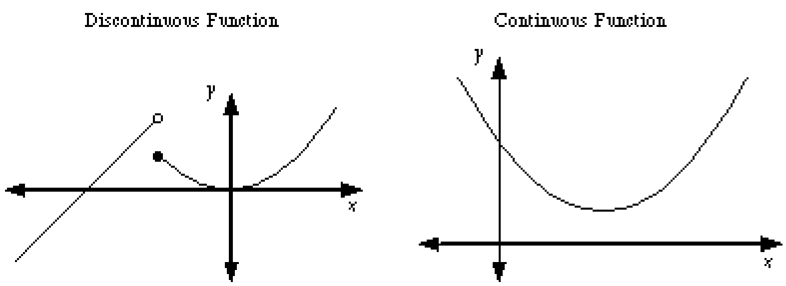
$$= 2y\frac{dy}{dy}$$

= 2y (1)

= 2 y

Q: Explain Discontinuous Function

A function with a graph that is not connected.



Using definition of Theorem 2.7.7, we can show that function is continuous at point c.

A function f is continuous from left at the point c if

1. f(c) is defined.

2. $\lim_{x \to c^-} f(x)$ exists.

3. $\lim_{x \to c^-} f(x)$ = f(c)

And f is called continuous from right at the point c if

1. f(c) defined.
2. $\lim_{x \to c^+} f(x)$ exists.
3. $\lim_{x \to c^+} f(x)$ = f(c)

If function f is continuous from both left and right at point c, then f is continuous at point x = c.

And generally function is continuous at x = c if

$\lim_{x \to c^+} f(x)$ **= f(c) =** $\lim_{x \to c^-} f(x)$

Q: Define Distance Formula

A point (x, y) moves so that its distance to (2, 0) is $\sqrt{2}$ times its distance to (0,1)

(a). Show that the point moves along a circle

(b). find the center and radius

Solution:

(a) By using distance formula

Distance of point to (2, 0) from (x, y) = $\sqrt{(x-2)^2+(y-0)^2}$

Distance of point to (0,1) from (x, y)= $\sqrt{(x-0)^2+(y-1)^2}$

By given condition

Distance to (2, 0) is $\sqrt{2}$ times its distance to (0,1)

So

$$\sqrt{(x-2)^2+(y-0)^2} = \sqrt{2}\,\sqrt{(x-0)^2+(y-1)^2}$$

Squaring both sides we get

$$\left((x-2)^2+(y-0)^2\right)=2\left((x-0)^2+(y-1)^2\right)$$

$$on \;\; simplification$$

$$x^2+4x+y^2-4y=2$$

As we done in above question, same procedure

$$By \; completing \; square$$

$$(x+2)^2+(y-2)^2=10$$

$$or \qquad (x+2)^2+(y-2)^2=\left(\sqrt{10}\right)^2$$

$$(x-(-2))^2+(y-2)^2=\left(\sqrt{10}\right)^2$$

Since this is the equation of the circle, and by the definition of circle, (The distance from a fixed point to a locus (movable point) is always constant)

It mean that the point (-2, 2) will always point on along the circle having the fixed radius $\sqrt{10}$

And

$$\Rightarrow center\ (-2,2)\ and\ radius\ \sqrt{10}$$

Q: Define Equity problem

Show that $|x^2+9| = x^2+9$

Solution:

$$\left|x^2+9\right| = x^2+9$$

$$As \quad |x| = a \qquad \Rightarrow \qquad -a \le x \le a$$

$$So \qquad -\left(x^2+9\right) \le x^2+9 \le x^2+9$$

$$-\left(x^2+9\right) \le x^2+9 \qquad and \qquad x^2+9 \le x^2+9$$

$$But\ second\ is\ true\ for\ all\ values\ so$$

$$-\left(x^2+9\right) \le x^2+9 \quad \Rightarrow \quad -2\left(x^2+9\right) \le 0 \quad ----(1)$$

$$\Rightarrow \quad x^2+9 \ge 0$$

$$\Rightarrow \quad x^2 \ge -9$$

Now it is clear that for all value of x , x^2 will be non-negative and hence always greater than -9. This shows that the inequality holds for all values of x and hence the whole set of real numbers is the solution set of the given inequality.

You may also use the test point method for that where you'll find two points -3 & 3 and hence the intervals (- infinity, -3) , (-3 , 3) & (3, infinity). All these intervals satisfy the inequality, also the points 3 and -3 do so which shows the set of real numbers is the solution set for the given inequality.

Q Explain Inequality problem

Solve the inequality and sketch the solution on a coordinate line.

2x-1>11x+9

Solution:-

2x-1>11x+9

2x-11x > 9+1

-9x > 10

Multiplying both sides by -

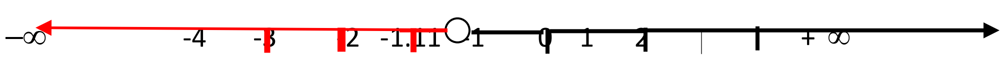
9x <- 10

$x < \frac{10}{9}$

x < -1.11

On coordinate line

Region A Region B



Or

We can draw on coordinate lien as

Verification of solution:

For region A,

Put x= -3, in equation (1)

we take this arbitrary point x=-3, because it lies in interval $-\infty$ and -10/9.

2(-3)-1 >11(-3) + 9

-6-1 > -33+9

-7 > -24 (True statement)

For region B,

Put x=0, in equation (1)

we take this arbitrary point x=0, because it lies in interval -10/9 and+∞.

2(0)-1>11(0) + 9

-1>9 (False statement)

Hence Solution Set is

$$\left(-\infty, -\frac{10}{9}\right)$$

Q:What is Inequality?

The statement involving < (less than), > (greater than) $\geq$ (greater than and equal to), $\leq$ (less than and equal to) is called inequality.

For example $3<4$ this statement (inequality) shows that the real number 3 is less than 4,

$3 \leq 4$ this inequality shows whether 3 is less or equal to 4. Here 3 is less than 4.

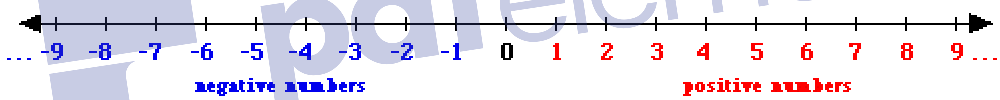
$4 \leq 4$ here 4 is equal to 4.

<u>Q: Define Integer Number</u>

Integers are the whole numbers, negative whole numbers, and zero.

For example, 43434235, 28, 2, 0, -28, and -3030 are integers, but numbers like 1/2, 4.00032, 2.5, Pi, and -9.90 are not.

We can say that an integer is in the set: {...3,-2,-1,0,1,2,3,...} It is often useful to think of the integers as points along a 'number line', like this:



Note that zero is neither positive nor negative

<u>Q: What is Interval:</u>

An **interval** is a set that contains a real number between two indicated real numbers numbers, and possibly the two numbers themselves.

For example $5 < x < 9$ is an interal in which the real number ***x*** lies between 5 and 9.

There are four types of interval

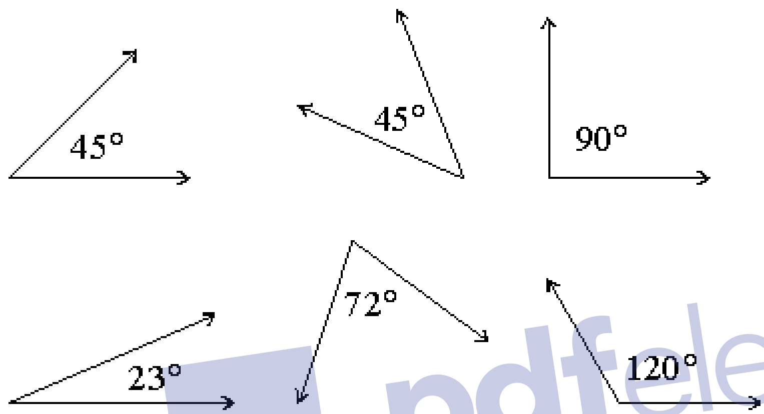
1. open interval
2. closed interval
3. semi open (one sided open)
4. semi closed (open sided closed)

Example Measure of Angle in degree

Degrees: Measuring Angles

We measure the size of an angle using degrees.

Example: Here are some examples of angles and their degree measurements.

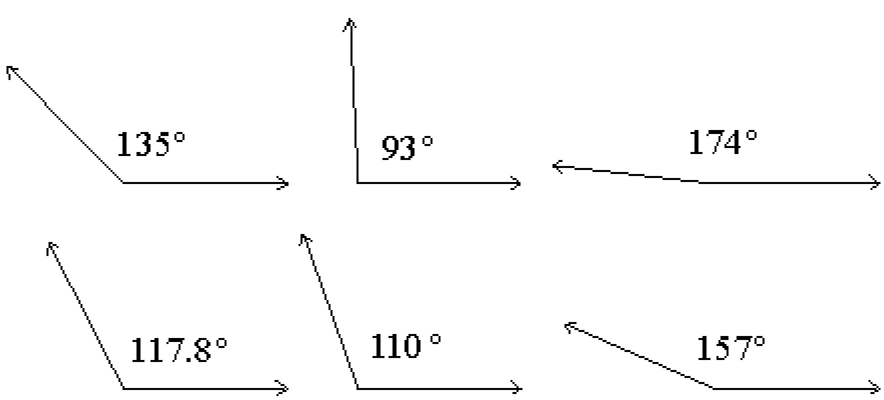


Q: Define Obtuse Angles

An obtuse angle is an angle measuring between 90 and 180 degrees.

Example:

The following angles are all obtuse.



Q:Define Open interval and closed Interval

Open interval

The open interval from a to b is denoted by (a, b) and is defined as $(a,b) = \{x : a < x < b\}$

Here the real numbers a and b are not included in the interval.

Closed interval:

The closed interval from a to b is denoted by [a, b] and is defined as $(a,b) = \{x : a \le x \le b\}$

Here the real numbers a and b are included in the interval.

<u>Q: Define Polynomial:</u>

The sum or difference of terms which have variables raised to positive integer powers and which have coefficients that may be real or complex

The following are all polynomials: $5x^3 - 2x^2 + x - 13$, $x^2y^3 + xy$, and $(1 + i)a^2 + ib^2$.

<u>Monomials:</u>

The polynomials with 1 term is known as monomials e.g x, xy^2, or x^2y^3z are monomials.

<u>Binomials:</u>

The polynomials with 2 terms which are not like terms is known as binomials. E.g. x + 2y, 2x – 3, $3x^5 + 8x^4$, and $2ab - 6a^2b^5$.

<u>Trinomials:</u>

A polynomial with three terms which are not like terms. The following are all trinomials: $x^2 + 2x - 3$, $3x^5 - 8x^4 + x^3$, and $a^2b + 13x + c$.

Standard form for a polynomial in one variable:

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

What is Rational number:

A *rational number* is any number that can be written as a ratio of two integers. In other words, a number is rational if we can write it as a fraction where the numerator and denominator are both integers and denominator can not be equal to zero.

The term "rational" comes from the word "ratio," because the rational numbers are the ones that can be written in the ratio form p/q where p and q are integers and q not equal to zero.

Every integer is a rational number, since each integer n can be written in the form n/1. For example 5 = 5/1 and thus 5 is a rational number. However, numbers like 1/2, 45, and -3/7 are also rational, since they are fractions whose numerator and denominator are integers and denominator not equal to zero.

Q: Explain Real Numbers

For any real number

$\sqrt{(a)^2}$ =|a|

Proof:

We can write a^2. In this way also

As

$a^2 = (+a)^2$

Also

$a^2 = (-a)^2$

Since $a^2 = (+a)^2 = (-a)^2$

The numbers +a and –a are squares roots of a^2.

As you know

$\sqrt{16}$ =+4 and -4

Or

$\sqrt{16} = \pm 4$

So we also write

$\sqrt{(a)^2} = \pm a$

Now there are two conditions

(1).

If a $\geq$ 0, then +a is the non negative square root of a^2 and

(2).

If a<0 then –a is the nonnegative square root of a^2.

Since $\sqrt{(a)^2}$ denotes the non negative square root of a^2

We have

Thus if we take

As from above (1)

$\sqrt{(a)^2}$ =+a if $a \geq 0$

As from above (2)

$\sqrt{(a)^2}$ = - a if a <0

That is

As

|a| =± a

We can write combine as

$$\sqrt{(a)^2} = \pm a$$

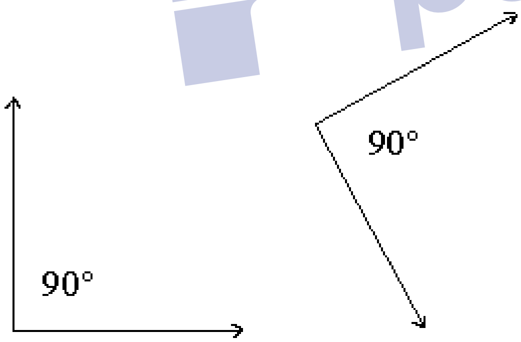
$$\sqrt{(a)^2} = |a|$$

Q: Define Right Angles

A right angle is an angle measuring 90 degrees. Two lines or line segments that meet at a right angle are said to be perpendicular. Note that any two right angles are supplementary angles (a right angle is its own angle supplement).

Example:

The following angles are both right angles.

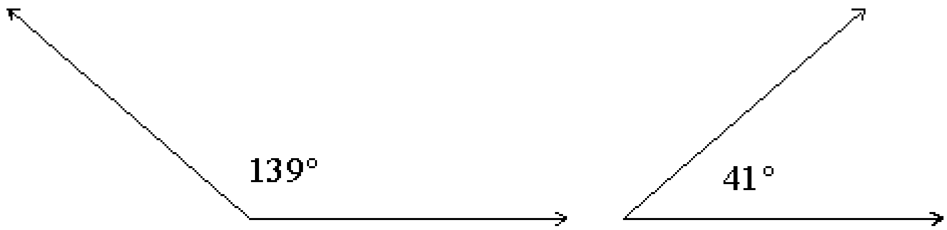


Q:Deifne Supplementary Angles

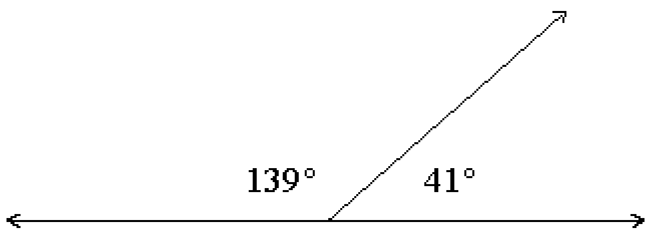
Two angles are called supplementary angles if the sum of their degree measurements equals 180 degrees. One of the supplementary angles is said to be the supplement of the other.

Example:

These two angles are supplementary.



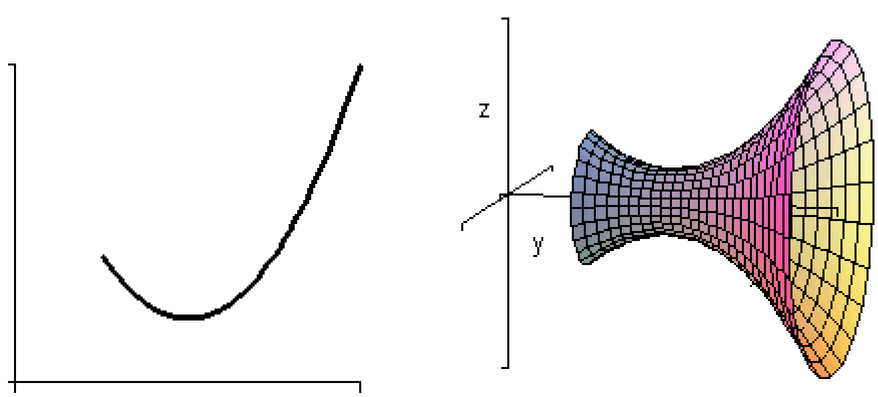
Note that these two angles can be "pasted" together to form a straight line!



Define Surface Area

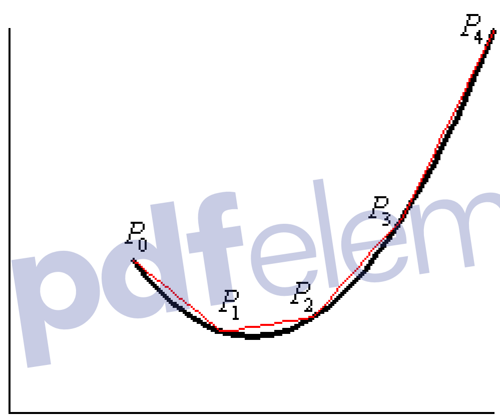
Surface Area

In this section we want to find the surface area of this region. Let us consider, rotating the continuous function $y = f(x)$ in the interval $[a, b]$ about the *x*-axis. Below is a sketch of a function and the solid of revolution we get by rotating the function about the *x*-axis.

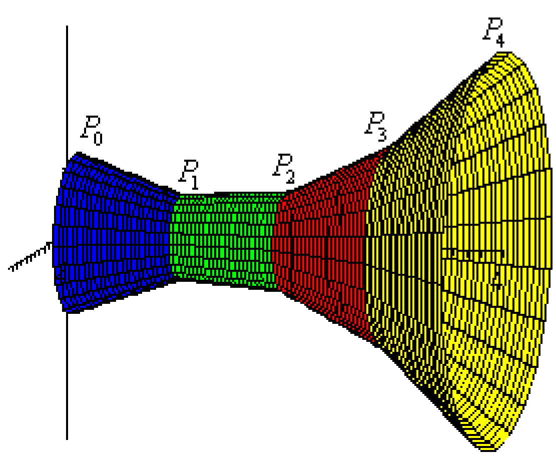


We can derive a formula for the surface area much as we derived the formula for arc length We will start by dividing the integral into *n* equal subintervals of width Δx. On each subinterval we will approximate the function with a straight line that agrees with the function at the endpoints of the each interval. Here is a sketch of that for our

Representative function using *n=4*.



Now, rotate the approximations about the *x*-axis and we get the following solid.



The approximation on each interval gives a distinct portion of the solid and to make this clear each portion is colored differently. Each of these portions is called frustums and we know how to find the surface area of frustums.

The surface area of a frustum is given by,

$$A=2\pi r l$$

Where,

$$r=\frac{1}{2}(r_1+r_2) \qquad r_1=\text{radius of right end}$$

$$r_2=\text{radius of left end}$$

l is the length of the slant of the frustum

Q: Difne Slope Formula

The Slope formula (Slope of the line between two points) between any two points A (x_1, y_1) and B (x_2, y_2) in the space is given by

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Where "m" is the slope of the line.

Example:

Find the slope of the line between the two points A (1, 1) and B (-2,-8)

Solution:

Slope formula between the two points is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Putting the values (as by the giving point s)

Thus $$m = \frac{-8-1}{-2-1} = 3$$

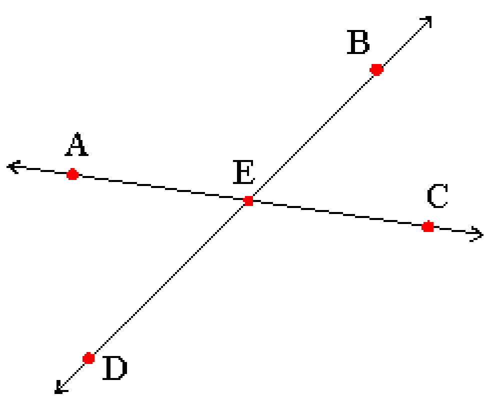
$$m = \frac{-9}{-3} = 3$$

$$m = 3$$

Thus slope of the line between the two points is $m = 3$

Q: Define Vertical Angles

For any two lines that meet, such as in the diagram below, angle AEB and angle DEC are called vertical angles. Vertical angles have the same degree measurement. Angle BEC and angle AED are also vertical angles.



Q.1# What is the difference between a Taylor series and a Maclaurin series?

Answer: The **Taylor series** is a representation of a function as an infinite sum of terms calculated from the values of its derivatives at a single point. It may be regarded as the limit of the Taylor polynomials. If the series is centered at zero, the series is also called a **Maclaurin series**.

Q.2# What are the applications of the Taylor series?

Answer:

- Some functions have no antiderivative which can be expressed in terms of familiar functions. This makes evaluating definite integrals of these functions difficult because the Fundamental Theorem of Calculus cannot be used. However, if we have a series representation of a function, we can oftentimes use that to evaluate a definite integral.

Example: Suppose we want to evaluate the definite integral

$$\int_0^1 \sin(x^2)\, dx$$

The integrand has no antiderivative expressible in terms of familiar functions. However, we know how to find its Taylor series: we know that

$$\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \dots$$

Now if we substitute $t = x^2$, we have

$$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{10!} - \frac{x^{14}}{14!} + \dots$$

In spite of the fact that we cannot antidifferentiate the function; we can antidifferentiate the Taylor series:

$$\begin{aligned}\int_0^1 \sin(x^2)\,dx &= \int_0^1 (x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \ldots)\,dx \\ &= (\frac{x^3}{3} - \frac{x^7}{7\cdot 3!} + \frac{x^{11}}{11\cdot 5!} - \frac{x^{15}}{15\cdot 7!} + \ldots)\Big|_0^1 \\ &= \frac{1}{3} - \frac{1}{7\cdot 3!} + \frac{1}{11\cdot 5!} - \frac{1}{15\cdot 7!} + \ldots\end{aligned}$$

Note that this is an alternating series so we know that it converges. If we add up the first four terms, the pattern becomes clear: the series converges to **0.31026.**

- Sometimes, a Taylor's series can tell us useful information about how a function behaves in an important part of its domain.
- Some differential equations cannot be solved in terms of familiar functions (just as some functions do not have antiderivatives which can be expressed in terms of familiar functions). In this case we use the Taylor's series.

Q.3# How do we find Maclaurin series of $f(x) = e^x$?

Answer: Maclaurin series are important in many areas of mathematics. They are often used to *define* functions.

To write down the Maclaurin series we need to know the value at $x = 0$ of every derivative of the function. This is usually the practical problem that we face in working out Taylor series. In this case it is easy since every derivative of e^x is e^x and this has value 1 at $x = 0$. It turns out that this is actually equal to the value of e^x for any value of *x.* So the Maclaurin series becomes

$$\boxed{e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots}$$

We often find this given as the *definition* of e^x.